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INVESTIGATION OF A CIRCULAR FLAT SLAB PRESTRESSED BY CONSTANT LENGTH CABLES

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INVESTIGATION OF A CIRCULAR FLAT SLAB PRESTRESSED BY CONSTANT LENGTH CABLES

A Project
Presented to the
Department of Civil Engineering Science
Brigham Young University

In Partial Fulfillment
of the Requirements for the Degree
Master of Civil Engineering

by
Kanaiyalal B. Shah
August, 1968

This project, by Kanaiyalal B. Shah, is accepted in its present form by the Department of Civil Engineering Science of Brigham Young University as satisfying the project requirement for the degree of Master of Civil Engineering.

May 21, 1968
Date

Arnold Wilson
Chairman, Advisory Committee

W. Ron Budge
Member, Advisory Committee

Clair S. Barton
Chairman, Major Department

This project is dedicated to my parents
whose quest and sacrifice for education
enabled me to reach this milestone

ACKNOWLEDGEMENT

The author wishes to express his deepest appreciation to Professor Arnold Wilson, Chairman of the Advisory Committee, for his guidance and stimulation in writing this project.

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b	Slab width
C_t	Distance from c.g.s. to top fibers
C_b	Distance from c.g.s. to bottom fibers
D	Over-all depth
d	Effective depth
f'_s	Ultimate strength of steel
f'_c	Ultimate strength of concrete
I	Moment of inertia
k_t	Top kern point
k_b	Bottom kern point
L	Panel width
n	Number of cables tangential to each circle
O	Center of circular slab
P	Effective prestressing force per wire
Z_t	Section modulus with respect to top fibers
Z_b	Section modulus with respect to bottom fibers

I. INTRODUCTION

A floating slab can be constructed as top slab on a large water tank. The roof slab floats on water when the water level is above the support level and rests on top of supports constructed within the water tank when the water level is below the support level.

Circular prestressed concrete floating slabs have been analyzed and constructed (4)* on the basis of a rectangular grid of reinforcement which required steel wires of different lengths. Attempts have not been made to analyze a floating prestressed concrete slab with wires of equal length or sets of equal length. Wires should be arranged in such a way that distances between crossings of cables would cause the cables to "pile-up" and prevent proper location of the prestressing forces. Wires can be arranged tangential to some arbitrary circle as seen in Fig. 1.

The purpose of this project was to investigate theoretically the behavior of a circular flat slab prestressed by equal length wires. The theoretical investigation includes arrangement for prestressing wires and stresses induced by each arrangement.

It is assumed that (i) the slab can float if superimposed load on slab is balanced by upward pressure of water, (ii) the connection between

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*Numbers in parentheses refer to references.

slab and vertical wall is rigid, (iii) the slab behaves similar to a flat slab when it rests on supports.

Methods of analysis are based on flat slabs (3) and on the behavior of prestressed concrete slabs (1).

The problem here is to investigate theoretically the behavior of a circular flat concrete slab 100 foot diameter prestressed by equal length wires.

II. THEORETICAL INVESTIGATION

Description of Slab

The slab is assumed to be a 100 foot diameter circular slab and the thickness being 4 inches. The slab is surrounded on the periphery by a vertical concrete wall 2 feet high and 4 inches thick. The slab is assumed to carry a live load of 40 p.s.f.

Concrete presumed to be used has an ultimate strength of 5,000 p.s.i. and unit weight of 150 p.c.f.

Prestressing cables presumed to be used are 1/2 inch diameter with an ultimate strength of 270 k.s.i.

Different number of cables, i.e., 9, 12, 15 and 18 are provided tangential to two arbitrary circles of 10 feet diameter and 40 feet diameter as shown in Fig. 1.

The slab is assumed to be supported when empty on short posts located 10 foot center to center throughout the roof area.

The theoretical investigation included determining stresses at typical points in a typical panel. The general analysis therefore consists of two parts: (a) stresses due to superimposed load, and (b) stresses due to prestressing forces. Stresses were determined when the slab was floating or resting on supports.

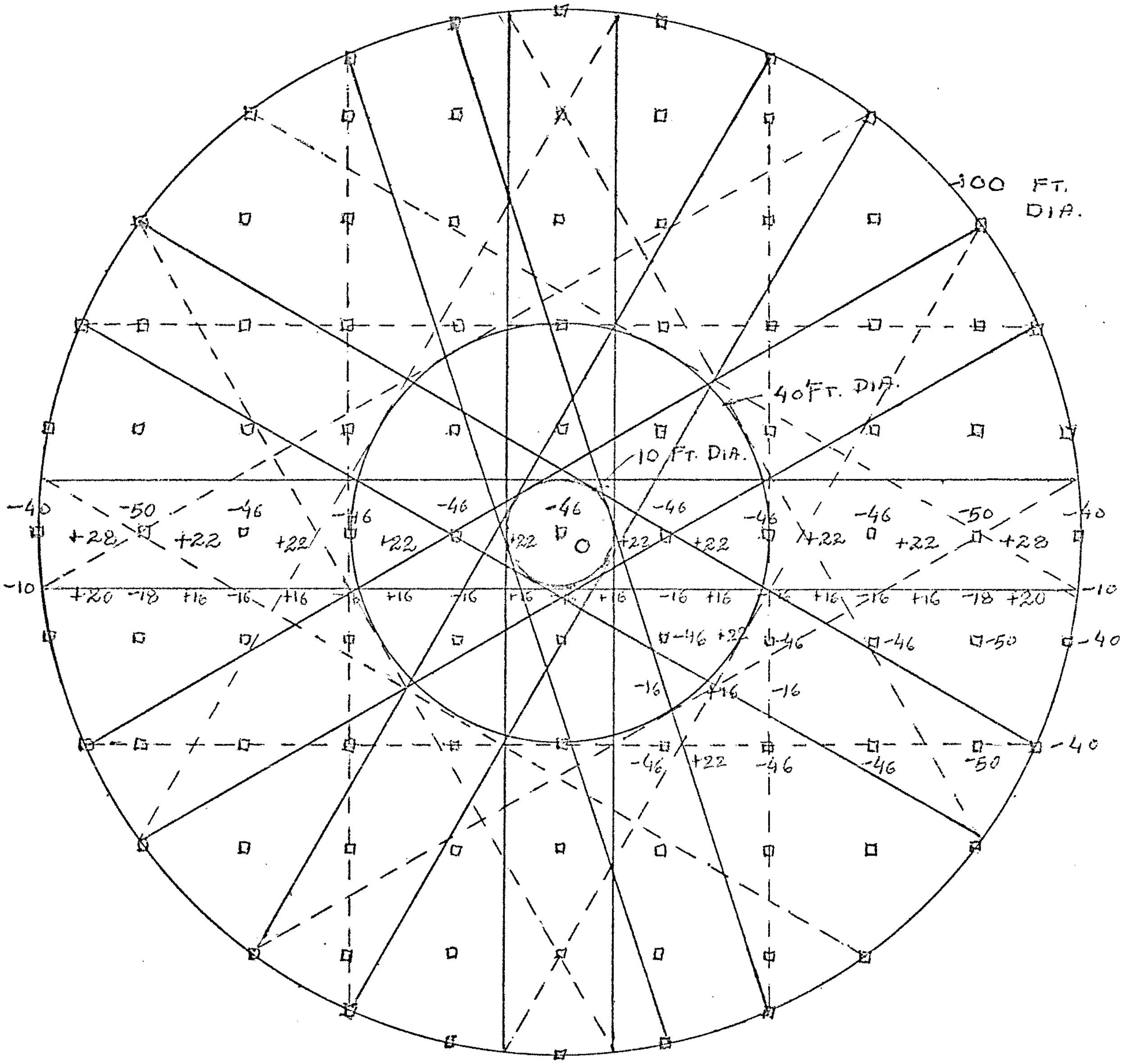


Figure 1 General Layout of Slab, Wires and Supports

Superimposed Load Investigation

Loads:

Live		40 p.s.f.
Weight of slab	$= \frac{(4 \times 12)\ 150}{144}$	50 p.s.f.
Equipment, etc.		5 p.s.f.
Total		95 p.s.f.

Cross section of support = 12 in. x 12 in. (assumed)

Moment coefficient (3) $M_o = 0.09\ WLF\ (1 - 2C/3L)^2$

$F = 1.15 - C/L$ But 1.00 $\quad C = 12$ inch

$= 1.15 - 1/10 \quad L = 10$ feet

$= 1.05$

W = Total live and dead load on panel

$= 95 \times 10 \times 10$

$= 9500$ lbs.

$M_o = 0.09 \times 9500 \times 10 \times 1.05\ (1 - 2/3 \times 1/10)^2$

$= 0.09 \times 9500 \times 10 \times 1.05 \times (14/15)^2$

$= 8120.0$ lb-ft.

Moment acting at various points in percentage of M_o (3) is shown in Fig. 1.

The position of a typical panel and typical points are shown in Fig. 2.

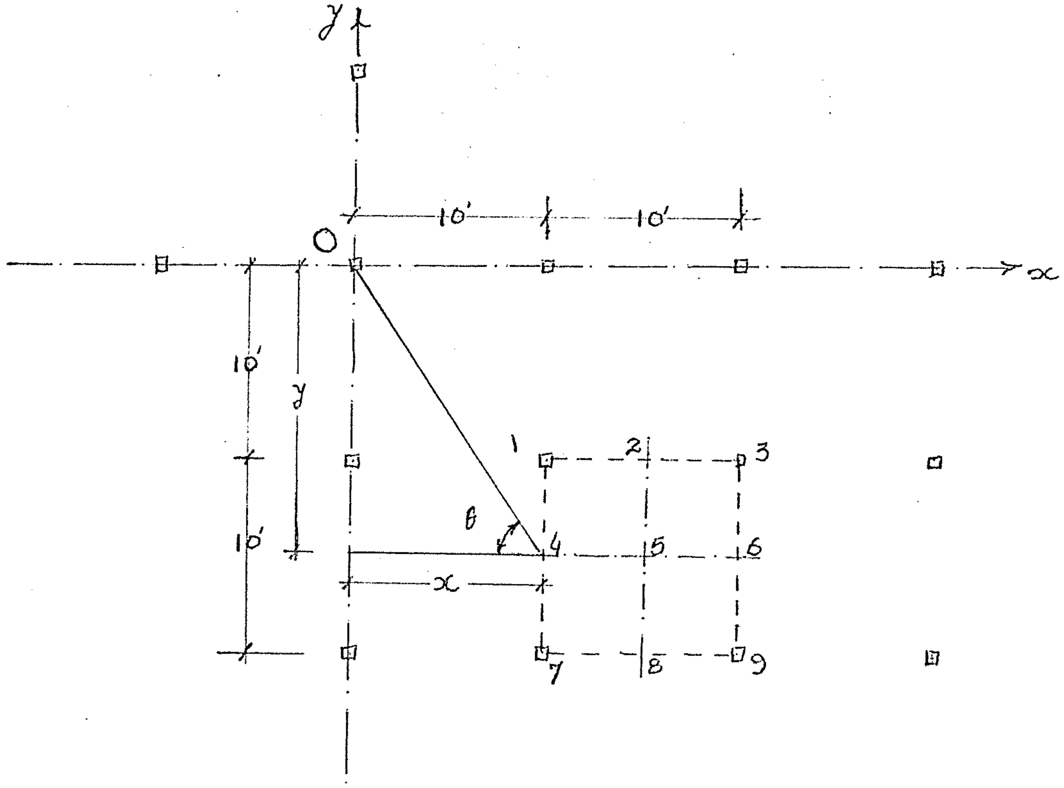
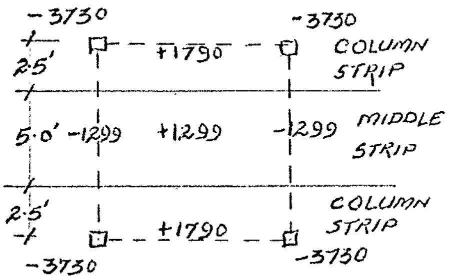
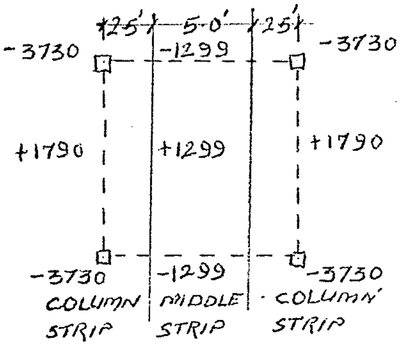


Fig. 2. Position of Typical Points

Moments at various points are calculated as:



Moment when strips are considered in x-direction (Moments in lb-ft.)



Moments when strips are considered in y-direction (Moments in lb-ft.)

Section Properties:

Depth D $= 4$ in.

Width of strip $b = 5$ ft. $= 60$ in.

Area $= 60 \times 4 = 240$ in.^{2}

$C_t = C_b = 2''$

$I = 1/12\ b\ D^3$

$= 1/12 \times 60 \times 4^3$

$= 320$ in^4

$Z_t, Z_b = I/C_b, C_t = 320/2 = 160$ in^3

$K_t, K_b = Z_t, Z_b/A = 160/240 = 0.66$ in.

Stress due to load at point 1 when strip considered in x-direction

$$= \frac{\text{Moment}}{Z_t, Z_b}$$

$$= \pm \frac{3,730 \times 12}{160}$$

$$= \pm 280 \text{ p.s.i.}$$

Stresses at other sections calculated and tabulated in Table IV.

Case 2

When the slab is floating, water exerts upward pressure which balances superimposed loads. The slab does not float on top of water, but at some depth which is required to balance the loads. Water exerts upward pressure of 62.4 p.s.f., 1 foot below water level. For load of 95 p.s.f. depth required is $95/62.4 = 1.523$ ft.

Prestressing Force Investigation

Minimum distances between crossings of cables for different numbers of cables are calculated and shown below.

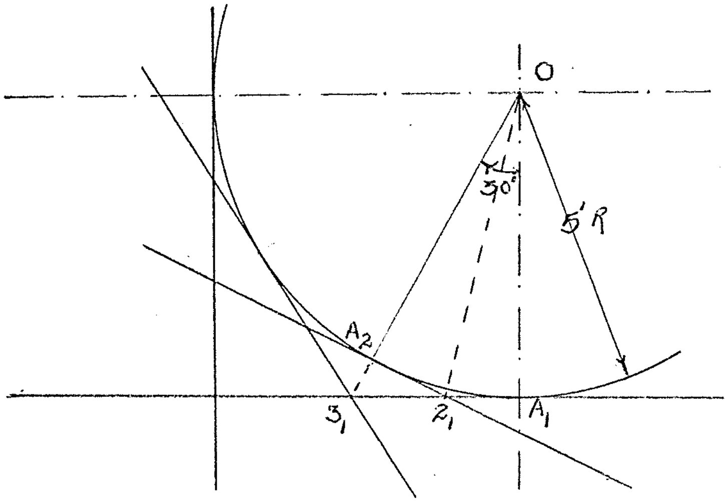


Fig. 3 Critical Distances Between Cable Crossings

The investigation is made for 12 cables tangential to a 10 ft. diameter circle. Normals to tangents make an angle of 30° with each other at the center of circle for 12 cables.

In triangle, $OA_1 2_1$

$\tan 30/2 = A_1 2_1 / OA_1$

$A_1 2_1 = 5 \times \tan 15^\circ$

$= 5 \times 0.2679 = 1.14$ ft.

In triangle, $OA_1 3_1$

$\cos 30^\circ = OA_1 / O3_1$

$= 5/03_1$

$O3_1 = 5/0.8660 = 5.78$ ft.

$A_2 3_1 = 5.78 - 5.00 = 0.78$ ft.

In the same triangle,

$$\tan 30^\circ = A_1 3_1 / OA_1$$

$$A_1 3_1 = 5 \times 0.5774$$

$$= 2.887 \text{ ft.}$$

$$3_1 2_1 = 2.887 - 1.140$$

$$= 1.747 \text{ ft.}$$

Above distances between cables are calculated and tabulated in the following table.

TABLE I

MINIMUM DISTANCES BETWEEN CABLE CROSSINGS

Number of Cables	$A_1 2_1$ ft.	$A_2 3_1$ ft.	$3_1 2_1$ ft.
9	1.82	1.528	2.375
12	1.14	0.780	1.747
15	1.063	0.477	1.163
18	0.881	0.320	0.939

Moment due to prestressing force.

No moment is created due to applied prestressing force as shown below. Therefore, bending stresses are not created. Stresses due to direct forces are created which are calculated later on.

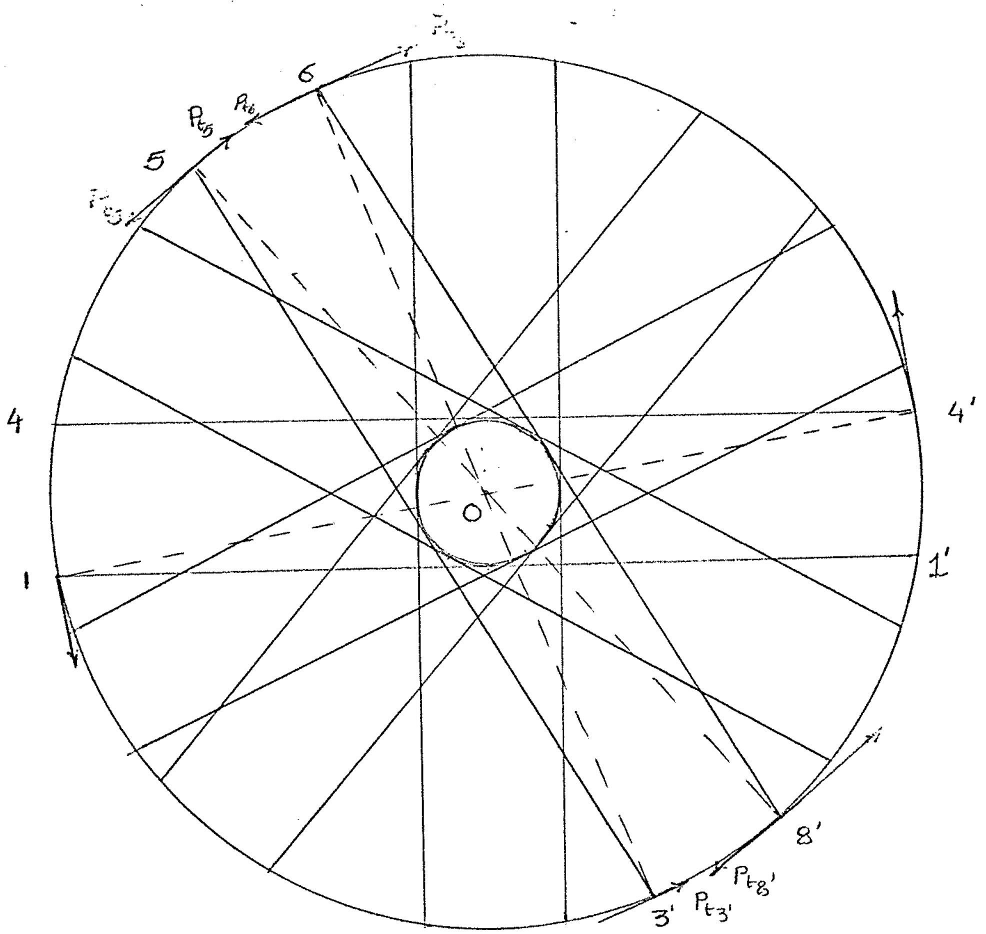


Fig. 4 Prestressing Forces

In the figure only 12 cables tangential to one circle are shown for simplicity. There are 24 ends of cables. Each cable is tangential to the inner circle. The consecutive points of the inner circle are at 30° measured from the center.

Consider two cables 5-5' and 6-6' which are symmetrical about a diagonal as shown in Fig. 4.

Consider points 5 and 6 of these two cables. There are tangential components P_{t5} and P_{t6} and radial components of prestressing force per unit length as shown where S is the spacing of the cables.

Now consider the other two diagonally opposite points 3' and 8' of cables 3-3' and 8-8' as shown in Fig. 4. These cables have also radial and tangential components P'_{t3} and P'_{t6} as shown.

The diameter of all cables is the same and the same amount of prestressing force is applied to all cables. Also angles intercepted at the center by radial lines and points 5, 6, 3' and 8'. Normals to lines joining center tangents to the inner circle joining points 5, 6, 3' and 8' shall be the same.

Therefore $P_{t5} = P_{t6} - P_{t3'} = P_{t8'}$ in magnitude, but their direction of application shall not be the same.

Considering the slab as a whole unit, the diagonally opposite forces, say acting tangential to points 5 and 8', (magnitude of each are the same) shall create a couple of magnitude = P_{t5} x diameter, (anti-clockwise), similarly, tangential forces acting at symmetrical points 6 and 5 will create a couple of magnitude = P_{t6} x diameter (clockwise).

The two forces P_{t5} and P_{t6} equal in magnitude make two couples of the same magnitude and opposite direction which balance each other.

Even though there are tangential components they do not create any circumferential moment, because equal and opposite forces balance each other.

Direct stresses due to Prestress

Prestressing forces and direct stresses are calculated for typical points on a panel as shown in Fig. 2.

Consider 270 k.s.i., one-half inch in diameter prestressing wires.

Area = $\pi/4\ (1/2)^2 = 0.197\ \text{in}^2$

Allowable stress in steel is 70% of the ultimate strength. Losses are considered to be 20%.

$$P = 0.197 \times (0.8 \times 0.7 \times 270)$$

$$= 29,200 \text{ lb.}$$

Direct stresses are induced due to tangential and radial components of prestressing force.

Let r_1 and r_2 = radii of inner and outer arbitrary circles to which wires are tangential.

r_1 = 5 ft. r_2 = 20 ft.

r = Radius of circle under consideration.

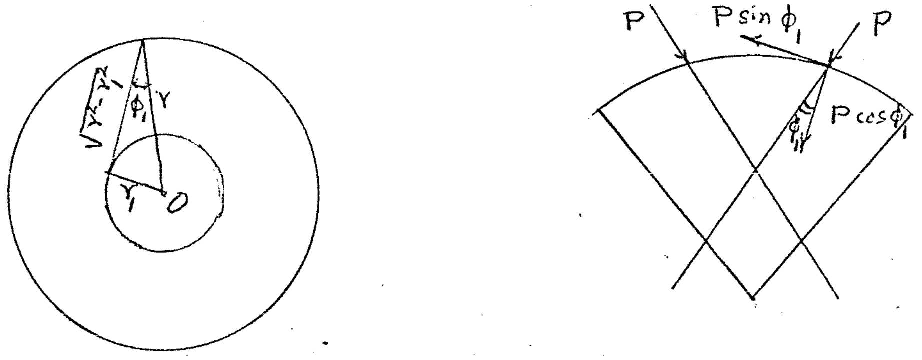


Fig. 5 Prestressing Force Components

$$\cos \phi_1 = \frac{\sqrt{\gamma^2 - r_1{}^2}}{r} \qquad \sin \phi_1 = \frac{r_1}{r}$$

Similarly, for two circles with radius γ_2 and γ ,

$$\cos \phi_2 = \frac{\sqrt{r^2 - r_2{}^2}}{r} \qquad \sin \phi_2 = \frac{r_2}{r}$$

There are two sets of equal length wires tangential to 10 ft. and 40 ft. diameter circles. Radial and tangential components of the prestressing forces shall be different for two regions: (a) between 10 ft. and 40 ft. diameter circles, since forces due to wires tangential to the 10 ft. diameter circle act, and (b) between 40 ft. and 100 ft. diameter circles, since forces due to wires tangential to both the 10 ft. and 40 ft. diameter circles act.

Between 10 ft. diameter and 40 ft. diameter circles

$$\text{Radial force} = 2\,n\,P \times \frac{\sqrt{r^2 - r_1{}^2}}{r}$$

$$\text{Tangential force} = 2\,n\,P\,\frac{r_1}{r}$$

Between 40 ft. and 100 ft. diameter circle

$$\begin{aligned}\text{Radial force} &= 2\,n\,P\,\cos \phi_1 + 2\,n\,P\cos \phi_2 \\ &= 2\,n\,P\,\left(\frac{\sqrt{r^2 - r_1{}^2}}{r} + \frac{\sqrt{r^2 - r_2{}^2}}{r}\right)\end{aligned}$$

$$\begin{aligned}\text{Tangential force} &= 2\,n\,P\sin \phi_1 + 2\,n\,P\sin \phi_2 \\ &= 2\,n\,P\left(\frac{r_1}{r} + \frac{r_2}{r}\right)\end{aligned}$$

Stresses act on an area circumscribed by radius r and thickness of the slab

$$\text{Radial or tangential stress} = \frac{\text{Radial or tangential force}}{2\,\pi\,r\,D}$$

Radial and tangential stresses are calculated at 10 ft., 40 ft., and 100 ft. diameter circles and tabulated below. There is no radial component at the 10 ft. diameter circle since all wires are tangential to this circle.

TABLE II

STRESSES DUE TO PRESTRESS AROUND CIRCLES

No. of Cables Tangential to Each Circle n	100 ft. dia. Circle		40 ft. dia. Circle		10 ft.dia. Circle
	Radial Stress	Tangential Stress	Radial Stress	Tangential Stress	Tangential Stress
9	66.2	18.0	84.2	109.0	349.0
12	88.0	23.8	112.0	145.0	475.0
15	110.0	29.8	140.0	181.0	579.0
18	132.4	36.0	168.4	218.0	698.0

NOTE: All stresses are compressive (negative) and in p.s.i.

Typical points, under consideration, are shown in Fig. 2.

Radius 0-1 $= \sqrt{x^2 + y^2}$

$= \sqrt{10^2 + 10^2}$

$= 14.12$ ft.

Radii of points 2 through 9 are calculated and tabulated in Table III.

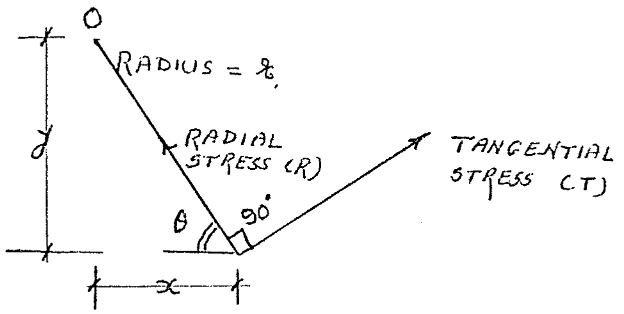
Stresses at typical cross-sections, as shown in Fig. 2, are calculated and are discussed in the chapter that follows.

III. RESULTS

The prestressing forces in wires create only direct stresses and they do not create any bending stresses. The direct forces act in the direction of wires and they create components in radial and tengential directions.

Radial and tangential components of the direct stresses are calculated at typical sections as shown in Fig. 2 and values are tabulated in Table III.

Stresses due to superimposed loads act in horizontal x and vertical y directions. Radial and tangential components of direct stresses due to prestress are resolved into x and y directions and superimposed on stresses due to loads. Results are tabulated in Table IV.



$\cos \theta = x/z$ $\qquad$ $\sin \theta = y/E$

Horizontal component of radial stress = $R \cos \theta$

Horizontal component of tangential stress = $T \sin \theta$

Horizontal stress = Difference of horizontal

TABLE III

DIRECT STRESSES AT TYPICAL SECTIONS

		9 Cables		12 Cables		15 Cables		18 Cables	
Pts.	Ft.	Radial Stress	Tangential Stress	Radial Stress	Tangential Stress	Radial Stress	Tangential Stress	Radial Stress	Tangential Stress
1	14.12	115.5	43.7	154.2	58.2	192.7	72.8	231.0	87.4
2	18.02	93.0	26.9	124.0	35.9	155.0	44.8	186.0	53.8
3	22.35	110.5	87.6	147.7	117.0	184.8	146.2	221.0	175.2
4	18.02	93.0	26.9	124.0	35.9	155.0	44.8	186.0	53.8
5	21.20	107.5	97.0	143.3	129.2	179.3	161.5	215.0	194.0
6	25.00	110.2	69.6	146.8	92.7	183.6	116.2	220.4	139.2
7	22.35	110.5	87.6	147.7	117.0	184.8	146.2	221.0	175.2
8	25.00	110.2	69.6	146.8	92.7	183.6	116.2	220.4	139.2
9	28.30	104.3	54.5	139.1	72.6	173.8	90.8	208.6	109.0

NOTE: All stresses are compressive (-ve) and in p.s.i.

components of radial and tangential stress

Vertical component of radial stress $= R \sin \theta$

Vertical component of tangential stress $= T \cos \theta$

Vertical stress = Sum of vertical components of radial and tangential stress

From Table IV, it can be concluded that 9 wires tangential to each circle are sufficient since stresses are well within allowable limits (allowable comp. stress = 2,250 p.s.i. and allowable tensile stress = 424 p.s.i.).

A cross section between 10 ft. and 40 ft. dia. circles is considered lying on circle with radius r, to find out adequate prestressing force.

$$\text{Radial stress } R = 2\,n\,P\,\frac{\sqrt{r^2 - r_2{}^2}}{r} \,/\, 2\pi\, r\, D$$

$$\text{Tangential stress } T = 2\,n\,P\,\frac{r_2}{r} \,/\, 2\pi\, r D$$

$$\text{Vertical component} = R \sin \theta + T \cos \theta$$

Since γ depends upon the position of section under consideration, it is difficult to find out any relationship which may be useful in determining the prestressing force just sufficient. A procedure of trial and error may be adopted.

TABLE IV

TOTAL STRESSES AT TYPICAL SECTIONS

					9 Wires at Each Circle			
			Stress Due to Loads		Stress Due to Prestress		Total of Load & Prestress Stress	
Pt.	Cos θ	Sin θ	x Dir. psi	y Dir. psi	x Dir. psi	y Dir. psi	x Dir. psi	y Dir. psi
1	0.707	0.707	±280.0	±280.0	-50.7	-112.5	+229.3 -330.7	+167.5 -392.5
2	0.833	0.555	∓134.0	± 97.5	-62.6	- 76.0	-196.6 + 71.4	+ 21.5 -173.5
3	0.895	0.447	±280.0	±280.0	-59.5	-127.7	+220.5 -339.5	+159.3 -407.7
4	0.555	0.833	± 97.5	∓134.0	-27.2	- 92.4	+ 70.3 -124.7	-266.4 + 41.6
5	0.707	0.707	∓ 97.5	∓ 97.5	- 7.4	-144.6	-104.9 + 90.1	-242.1 -147.1
6	0.800	0.600	± 97.5	∓134.0	-46.5	-121.8	+ 41.0 -144.0	-255.8 + 12.2
7	0.447	0.895	±280.0	±280.0	-29.1	-137.9	+250.9 -309.1	+142.1 -417.9
8	0.600	0.800	∓134.0	± 97.5	-10.6	-129.9	-144.6 +123.4	- 32.4 -227.4
9	0.707	0.707	±280.0	±280.0	-35.3	-112.0	+244.7 -315.3	+168.0 -392.0

TABLE IV (continued)

	12 Wires at each Circle				15 Wires at each Circle			
	Stress Due to Prestress		Total of Load & Prestress Stress		Stress Due to Prestress		Total of Load & Prestress Stress	
Pt.	x Dir. psi	y Dir. psi	x Dir. psi	y Dir. psi	x Dir. psi	y Dir. psi	x Dir. psi	y Dir. psi
1	-67.8	-150.0	+212.2 -347.8	+130.0 -430.0	- 84.9	-187.5	+195.1 -364.9	+ 92.5 -467.5
2	-83.4	- 98.7	-217.4 + 50.6	- 1.2 -196.2	-104.3	-122.9	-238.3 + 29.7	- 25.4 -200.4
3	-79.7	-170.4	+200.3 -359.7	+ 99.6 -450.4	-100.0	-213.5	+180.0 -380.0	+ 66.5 -493.5
4	-38.9	-123.2	+ 58.6 -136.4	-257.2 + 10.8	- 48.1	-154.1	+ 49.4 -145.6	-288.1 - 20.1
5	-10.0	-192.7	-107.5 + 87.5	-290.2 - 95.5	- 12.7	-241.0	-110.2 + 84.8	-338.5 -143.5
6	-61.7	-152.1	+ 35.8 -159.2	-286.1 - 18.1	- 77.1	-223.0	+ 20.4 -174.6	-353.0 - 89.0
7	-38.8	-184.3	+241.2 -338.8	+ 95.7 -464.3	- 48.5	-230.4	+231.5 -428.5	+ 49.6 -510.4
8	-13.9	-172.9	-147.9 +120.1	- 75.4 -270.4	- 17.0	-216.3	-151.0 +117.0	-118.8 -313.8
9	-47.0	-149.5	+233.0 -327.0	+130.5 -429.5	- 58.6	-187.2	+221.4 -338.6	+ 92.8 -467.2

TABLE IV (continued)

	18 Wires at each Circle			
	Stress Due to Prestress		Total of Load & Prestress Stress	
Pt.	x Dir. psi	y Dir. psi	x Dir. psi	y Dir. psi
1	-101.5	-225.0	+178.5 -381.5	+ 56.0 -505.0
2	-125.1	-148.1	-259.1 + 8.9	- 50.6 -245.6
3	-120.6	-255.0	+159.4 -400.6	+ 25.0 -535.0
4	- 58.3	-184.9	+ 39.2 -155.8	-318.9 - 50.9
5	- 14.8	-289.0	-112.3 + 82.7	-336.5 -191.5
6	- 92.7	-243.5	+ 4.8 -190.2	-377.5 -109.5
7	- 58.0	-274.4	+222.0 -338.0	+ 5.4 -554.4
8	- 20.9	-259.7	-154.9 +113.1	-162.2 -357.2
9	- 70.6	-224.0	+209.4 -350.6	+ 56.0 -504.0

IV. DISCUSSION

The floating slab can be advantageously designed and utilized if the dead and live loads are kept at minimum. Minimum loads allow the slab to float at the minimum depth in water.

The configuration of prestressing wires was not located tangential to only one circle, but was tangential to two circles. Such configuration helps in keeping the stress concentration low, even though stresses were high around circles to which wires were tangential. Also, location of wires tangential to two circles is helpful in providing sufficient room between crossings of wires so that construction might not be difficult.

The results of the stress analysis indicated that stresses were critical when the slab was resting on supports and not when it was floating. Loads are balanced when the slab is floating. Prestressing forces created only direct stresses even when it was floating. Superposition of stresses due to dead and live loads on the stresses due to the prestressing force also indicated that stresses were critical when the slab was resting on supports.

Bending due to concentrated load on the periphery of the slab caused by the vertical wall was neglected.

Within the inner arbitrary circle no prestressing wire was provided and hence it is recommended to keep the diameter of the circle to a minimum to avoid adverse stress conditions within the circle. As an alternate it is recommended to run a cable or two through the circle.

The author is satisfied with the results found by the method adopted, except that stresses were averaged in presuming that radial and tangential forces act on the whole circle. This assumption gave average results.

V. CONCLUSIONS

A circular concrete flat slab prestressed by equal length cables was analyzed. The advantages of such a slab are that they require equal length wires an hence cutting of wires in different lengths can be avoided and prefabrication of wires becomes possible.

Stresses for different number of wires indicated that 9 wires, with assumed prestressing force and cross section area, tangential to each circle, were sufficient for the assumed loadings.

Attempts to find a generalized form for finding adequate prestressing forces were not successful. The method of trial and error is suggested.

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An Abstract of a Project
Presented to the
Department of Civil Engineering Science
Brigham Young University

In Partial Fulfillment
of the Requirements for the Degree
Master of Civil Engineering

by
Kanaiyalal B. Shah
August, 1968

ABSTRACT

The purpose of this project was to investigate theoretically the behavior of a circular flat slab prestressed by equal length wires.

A circular slab was analyzed and stresses due to various combinations of wires were calculated. The slab was designed for a particular loading.

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