Utilizing Genetic Optimization on Buckling Restrained Braced Frames to Evaluate the Accuracy of the Equivalent Lateral Force Method

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ABSTRACT

Utilizing Genetic Optimization on Buckling Restrained Braced Frames to Evaluate the Accuracy of the Equivalent Lateral Force Distribution

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A study was performed to determine the optimal brace distribution for tall buckling restrained braced frames under lateral seismic loading. Optimal designs were found using a genetic algorithm in the open source software known as OpenSees. The optimum designs were compared and a distribution of braces sizes was determined that can be used in place of the method known as the equivalent lateral force procedure. The distributions behave better than the ELF procedure design by lowering the story drift and reducing the total core area of all the braces in the structure. Nine, twelve and eighteen story structures were used in this study.

Keywords: Kyle Scott Atwood, genetic optimization, seismic, ELF

ACKNOWLEDGEMENTS

This work was possible due to the support from Dr. Paul W. Richards in the Civil and Environmental Engineering department in the Ira A. Fulton College of Engineering and Technology at Brigham Young University. Lukas Balling, Graham Oxborrow, Eric Hoffman and Chris Yeates provided substantial contributions to the genetic algorithm used and programmed in OpenSees as part of previous research. I would also like to thank Dr. Richard J. Balling, and Dr. Fernando S. Fonseca for all of the help and knowledge that they have provided to me. Finally I would like to thank my wife, Stephanie, for the encouragement and support extended to me throughout the duration of this research.

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1 INTRODUCTION

Where earthquake and wind loads occur, lateral force resisting systems are required to support a structure. These lateral force resisting systems in steel buildings commonly consist of moment frames or braced frames. Moment frames rely on stiffness from the connections and members in the frame to resist forces; they provide ductility, while also removing unsightly diagonal braces. Braced frames use a diagonal member to resist lateral forces and are more economical and stiff. An increasingly popular and efficient system is that of buckling restrained braces (BRBs). A BRB comprises of a steel core encased by an outer shell that resists global buckling; this design allows the core to deform inelastically in both tension and compression under high lateral loading while retaining its strength.

The design of these lateral force resisting systems can be accomplished by time history analysis. Time history analysis looks at a previously recorded ground motion, and utilizing the numerical integration of the equation of motion, calculates the response of a structure to that specific ground motion. Nonlinear time history analysis provides maximum displacements along with all other displacements as the building is subjected to ground motion (Hamburger, 2009).

The leading variable when designing a buckling restrained braced frame (BRBF) is the cross sectional area of the brace's core. The core area drives the cost of a BRB and therefore should be reduced to what is needed to ensure a safe structure. These areas can be reduced to their optimum by using genetic algorithms. Genetic algorithms are used to find optimum designs by implementing the evolutionary process. Using the information obtained from a nonlinear time history analysis in a genetic algorithm can lead to the optimization of a structure’s lateral force resisting system under previously recorded earthquakes.

A common method for determining the size of the core areas required is known as the equivalent lateral force method (ELF). This method is well known and often used due to its ease of computation. This method calculates the total base shear of a structure and then, based on floor weights and heights, distributes a portion of that base shear force to each level of the structure according to code set forth in ASCE-7 (2010). This paper presents a method for distributing lateral forces on a taller structure in order to minimize story drift while reducing total brace area. This is to be accomplished by obtaining optimum designs from nonlinear time history analysis performed within a genetic algorithm. The distribution will be verified under suites of earthquakes to determine the accuracy of the method.

2 PREVIOUS RESEARCH

This research is a continuation of previous findings in this field. Balling, Balling and Richards (2009) presents a new method for the design of buckling-restrained braced frames that utilizes a genetic optimization algorithm and time history analysis. Their method is compared to the equivalent lateral force procedure and their results are embodied in design curves that can be used for quick design. Their work was focused on 1-, 3- and 5-story buildings and they found that the optimum variation is mostly linear which is not predicted by the equivalent lateral force procedure.

Time history analysis performed within a genetic algorithm optimization program takes a considerable large amount of time and may not converge to the true optimum each time. Hoffman and Richards (2013) presents a study that evaluates different techniques implemented in genetic optimization algorithms and the results they produce. They used a nine story buckling restrained braced frame that was being optimized to minimize brace area under seismic loading. The optimization program used nonlinear time history analysis as its tool to determine the efficiency of a design. It was shown that the true optimum was not converged to using a typical genetic algorithm. Implementing forced diversity, mutation and cross over led to the convergence to the same optimum regardless of the initial design and overall time to reach this optimum was reduced.

Oxborrow G. (2009) optimizes 3-, 6-, 9-, 12- and 18-story buildings comprising of a BRBF design and compares the optimized distribution of brace areas at each floor to the distribution reached when utilizing the ELF method. The optimization program was developed and refined by the two studies mentioned before this one in a software program entitled OpenSEES developed by the Pacific Earthquake Engineering Research Center.

Weight of steel drives the cost of a steel building and as such brace areas are often optimized in order to reduce them; however, other factors exist in reducing the overall weight of a steel structure. Yeates C. expanded to work completed by the previous three studies and considered multiple design objectives in the optimization program. Brace area along with column demands were optimized in order to reduce the necessary steel required to satisfy drift restraints. It was found that by minimizing brace areas, base column demands were minimized but brace areas cannot be minimized by minimizing base column demands.

3 OPTIMIZATION PHASE

3.1 Optimization Results

The goal for the optimization phase of this project was to find the optimum designs of BRBFs under particular ground motions and identify the trends and consistencies in order to implement these in a new distribution of forces similar to the ELF method. Three earthquake ground motion records were used to optimize the brace areas of 9-, 12- and 18-story buildings. The optimization was accomplished within the OpenSees software and the source code was developed in conjunction with the studies mentioned in the “Previous Research” portion of this paper. An exhaustive explanation of this optimization program has been addressed previously and therefore will not be addressed in this paper.

The three ground motions considered in this part of the study were the 1979 Imperial Valley (El Centro), 1989 Loma Prieta (Hollister City Hall) and 1994 Northridge (Canoga Park) records. The building parameters in addition to the three heights were; 100 feet by 100 feet building footprint, 15 feet tall and 25 feet wide bays and a single column of bays on each side of the building contained a single BRB in each bay.

The genetic algorithm produces the top 100 optimal designs. The top five designs were considered and the average brace area at each level for the three different heights and ground motions were plotted. In order to provide a more balanced perspective of these optimum designs, the normalized values were averaged and plotted as well.

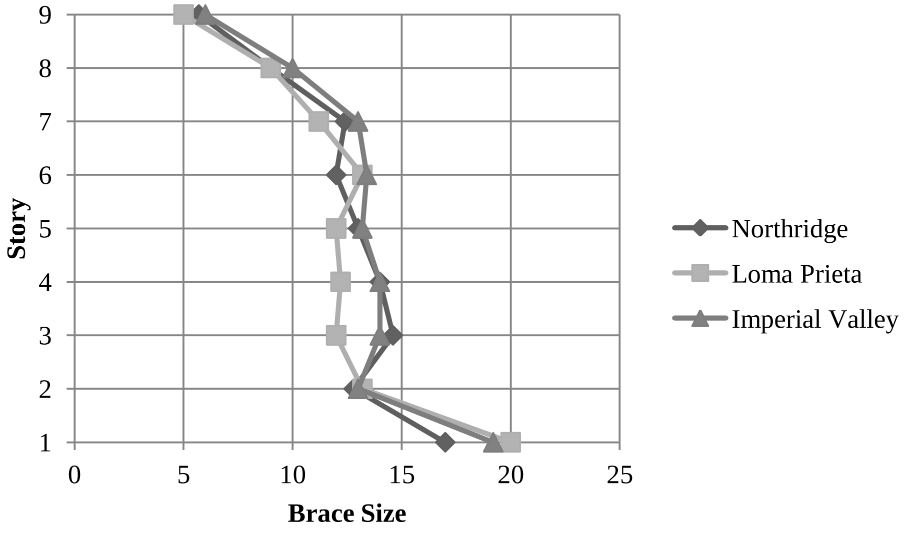

Figure 3-1: Average of the top five optimized brace areas for 9-stories.

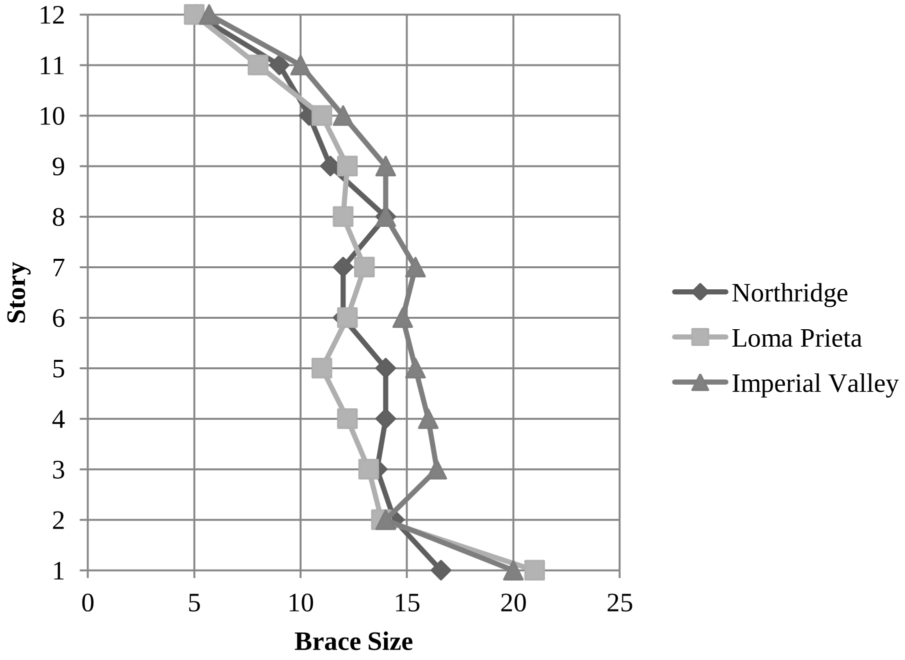

Figure 3-2: Average of the top five optimized brace areas for 12-stories.

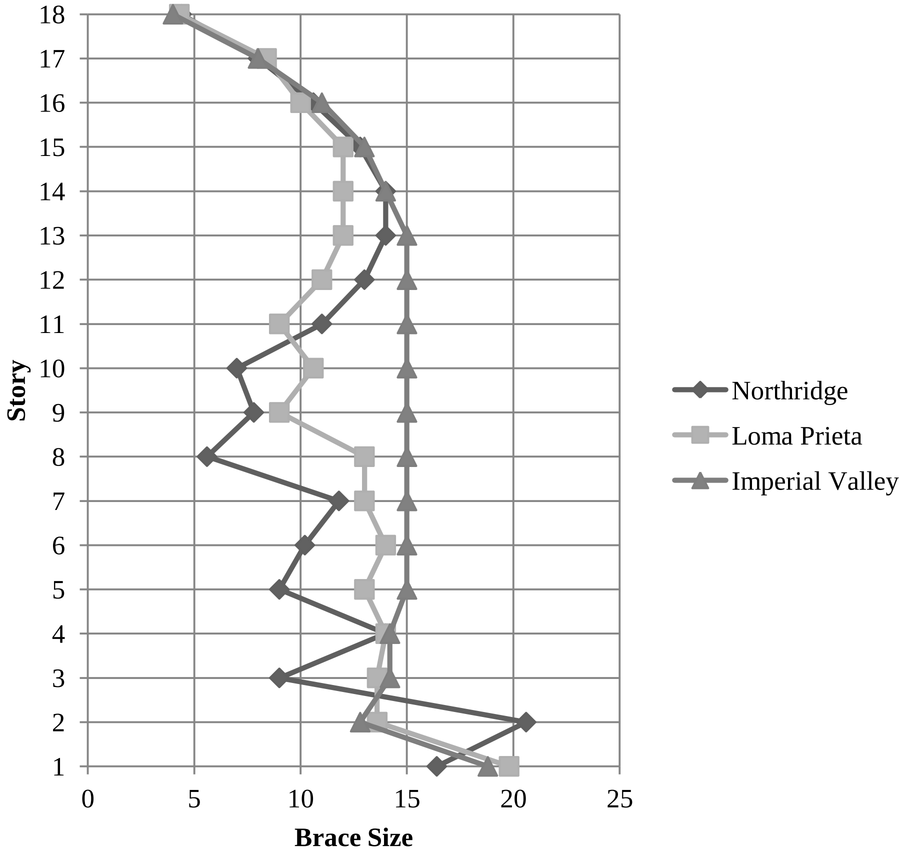

Figure 3-3: Average of the top five optimized brace areas for 18-stories.

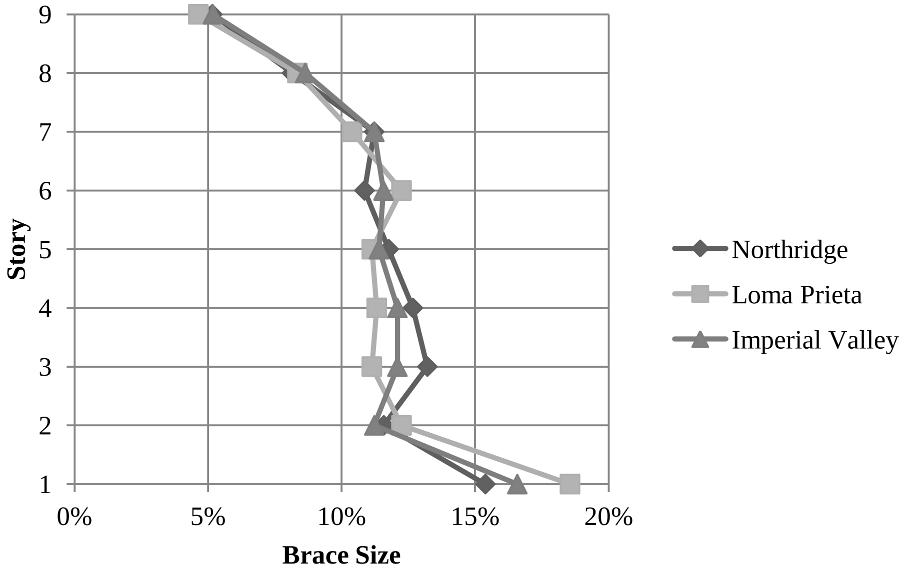

Figure 3-4: Normalized average of the top five braced designs for 9-stories.

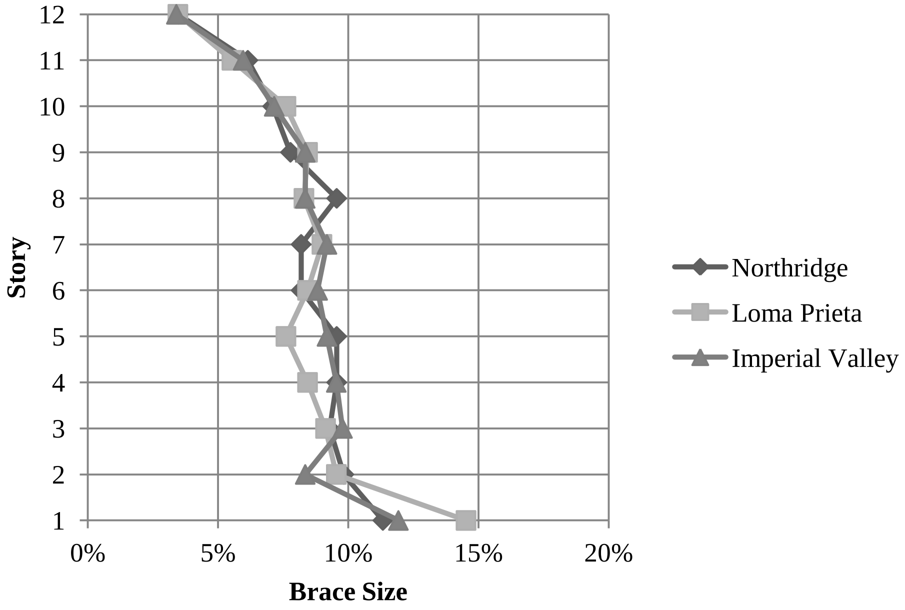

Figure 3-5: Normalized average of top five brace designs for 12-stories.

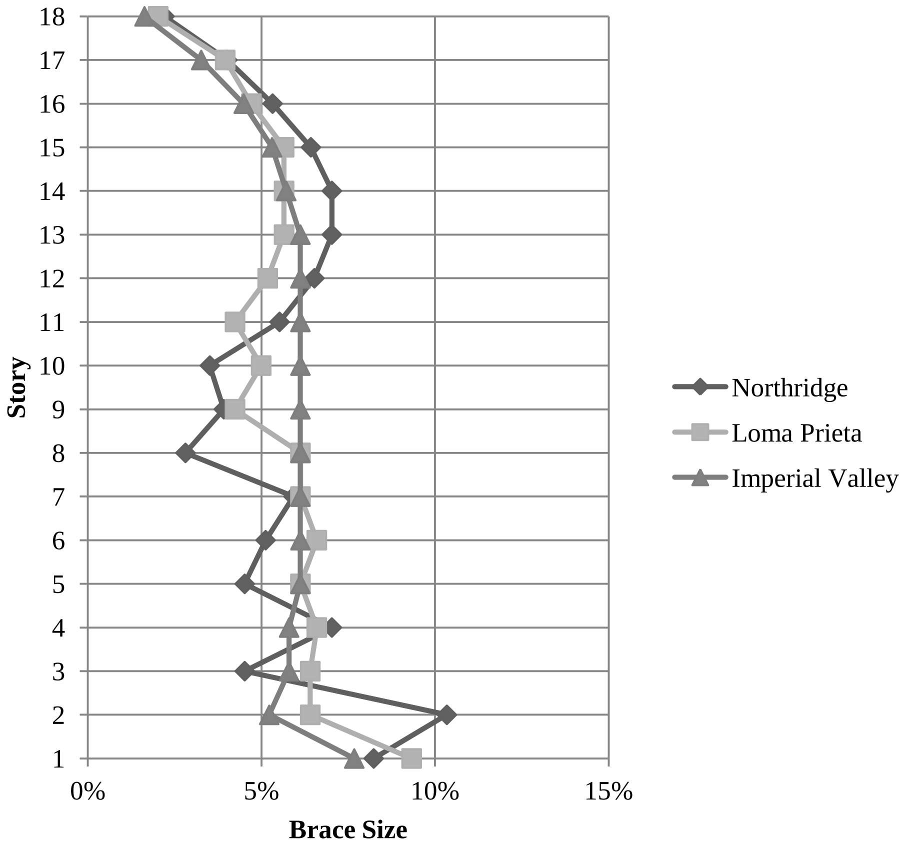

Figure 3-6: Normalized average of top five brace designs for 18-stories.

3.2 Equivalent Lateral Force Method

The equivalent lateral force method is the current design practice for preliminary and often final design for lateral force resisting systems. In order to provide a comparison to these optimum designs, the ELF method was applied to the three different story heights and brace areas were obtained.

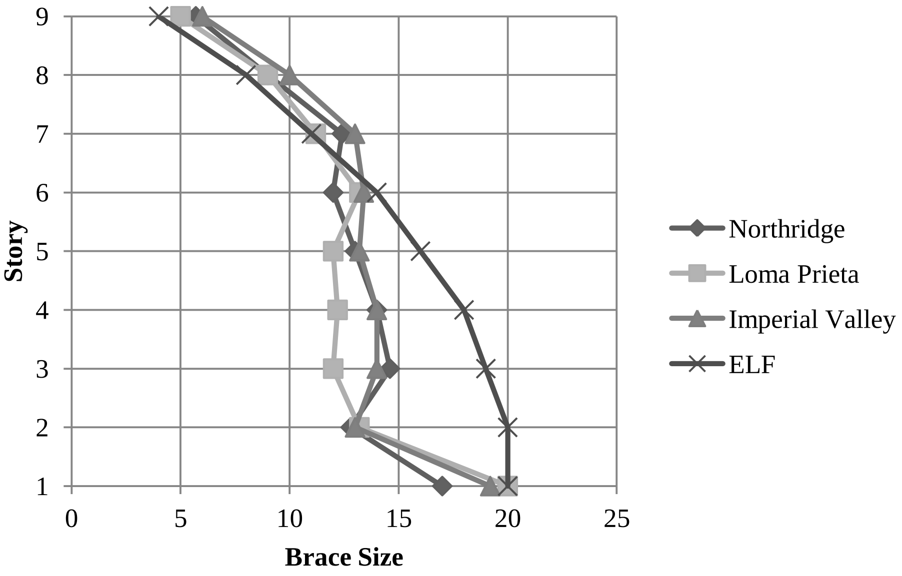

Figure 3-7: Brace areas for all three ground motions and the ELF method for 9-stories.

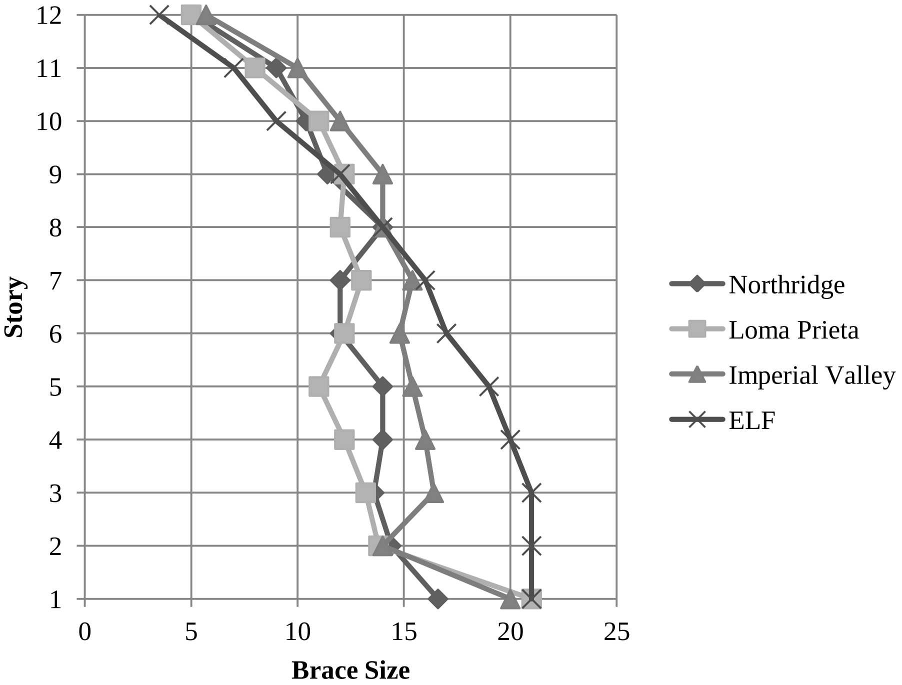

Figure 3-8: Brace areas for all three ground motions and the ELF method for 12-stories.

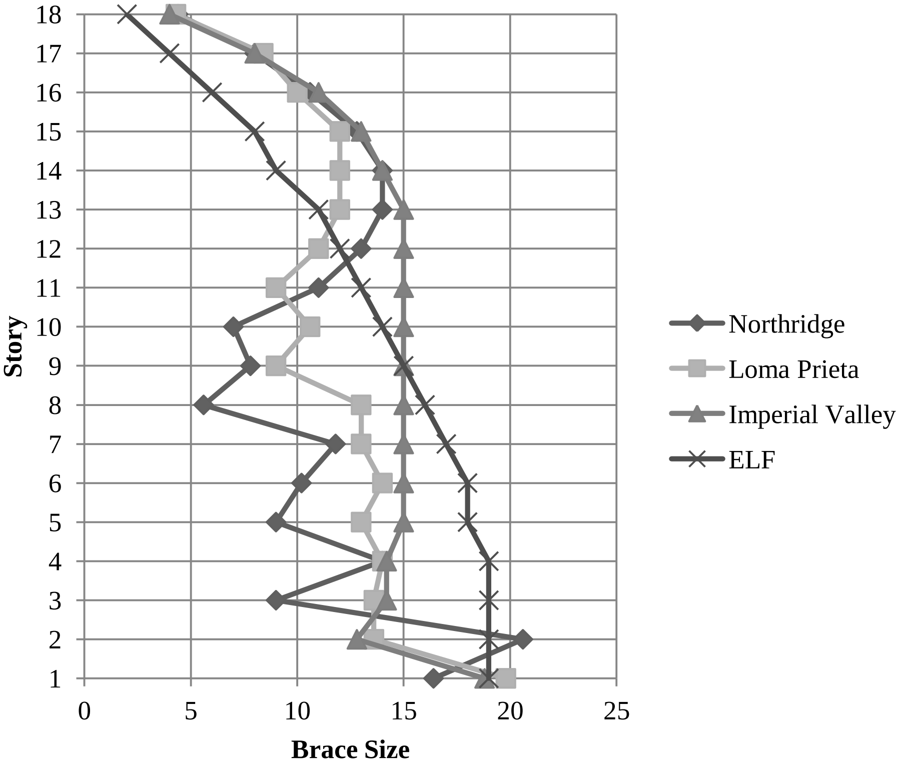

Figure 3-9: Brace areas for all three ground motions and the ELF method for 18-stories.

4 DISTRIBUTION

The ELF method calculates a base shear force for the entire building. The force of the base shear is a function of many factors but some of the more notable ones are location, weight, height and the type of lateral force resisting system. This base shear is divided and then distributed to each story of the building; the magnitude of the force depends upon the height and weight of the floor in question relative to the overall height and weight of the building.

Comparative to the ELF method, two distributions were developed in part of this research. The first is based off of the actual brace areas found from the optimization phase of this project while the second is a result from the behavior of the normalized areas. The first distribution is called the Atwood Richards Method (ARM) and the second is called Distribution 1 (DIST). Each distribution is based off of the lowest level brace size, which must have enough capacity to meet the demands of the calculated base shear of the building. The base shear is calculated the same way as prescribed in ASCE-7 (2010). The ARM contains a zigzag pattern for the lower story braces, holds constant in the middle and slowly tapers down in the highest stories. DIST is a simpler design, brace areas drop after the initial brace size and remain constant for three stories and then the size is reduced again and the highest stories have the same distribution as the ARM (see Table 4-1).

Table 4-1: Brace size distribution based on the first story brace size.

		Story								
		1	**2**	**3**	**4**	**5**	**6***	**7**	**8**	**9**
Method	DIST	n	n-3	n-3	n-3	n-3	.667n	.583n	.5n	.33n
	ARM	n	n-5	n-2	n-5	n-3	.667n	.583n	.5n	.33n

*This story is repeated as the number of stories increases.

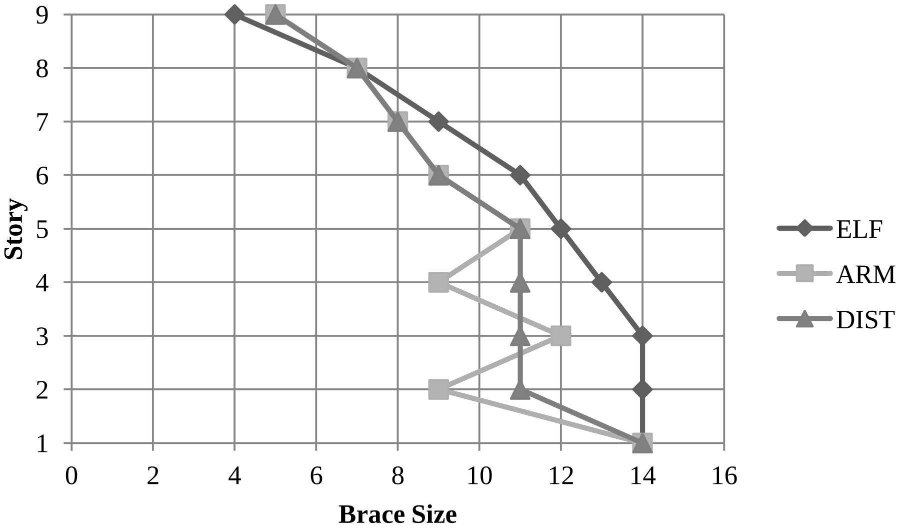

Figure 4-1: Brace sizes of the different distributions for 9-stories.

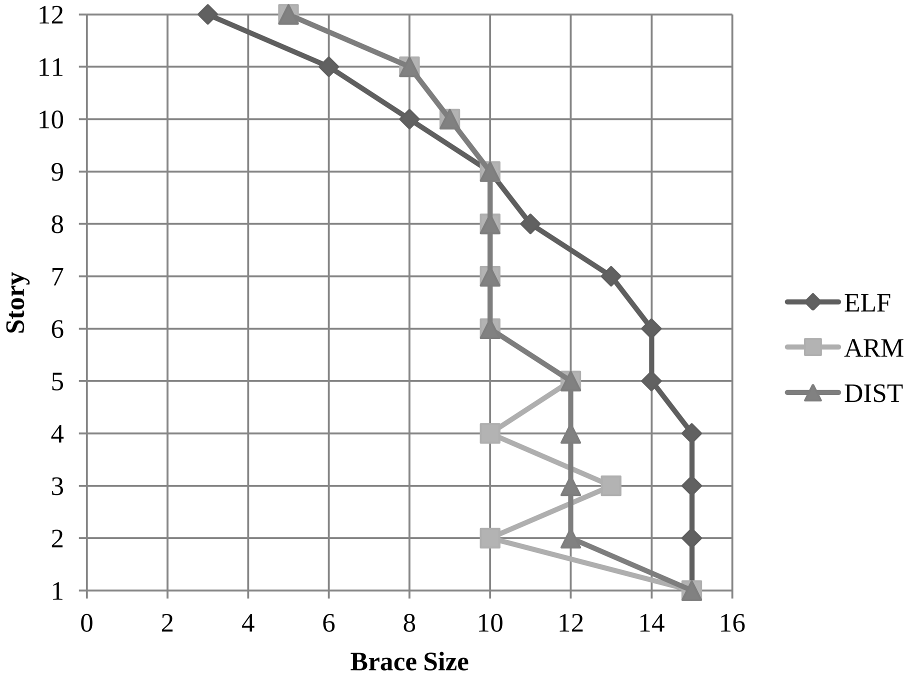

Figure 4-2: Brace sizes of the different distributions for 12-stories.

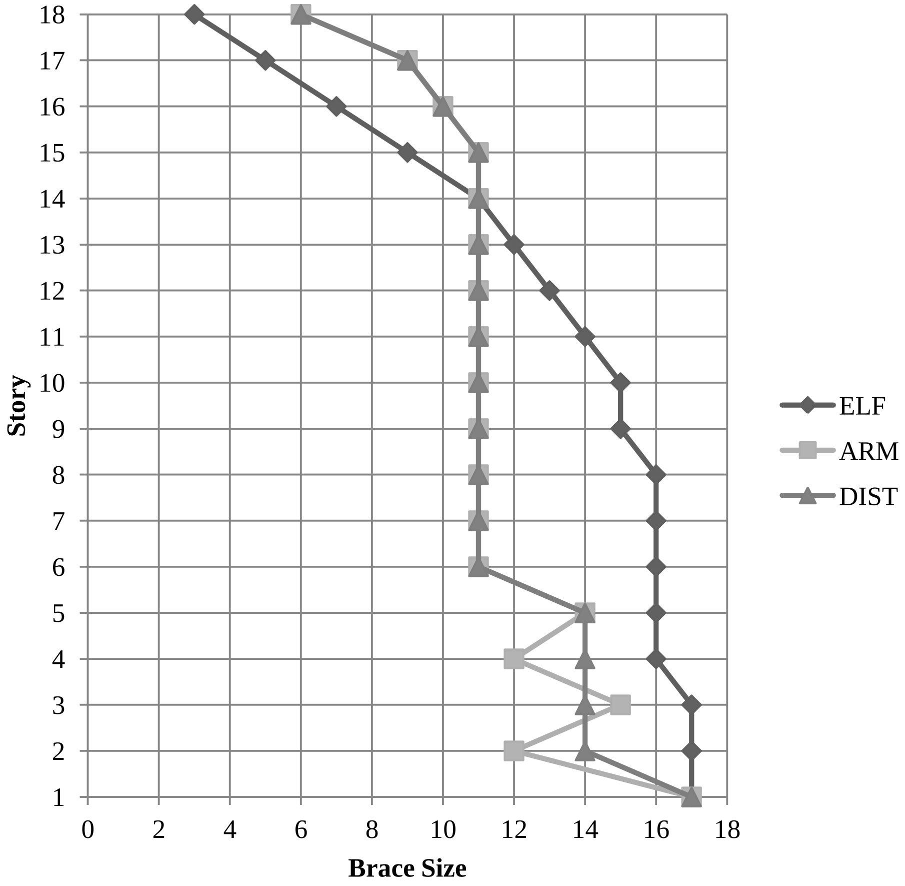

Figure 4-3: Brace sizes of the different distributions for 18-stories.

The ARM and DIST distributions provide brace area savings from 10% up to 14% when compared to the brace areas calculated by the ELF method. This reduces the weight and cost of these braced frames.

Table 4-2: Brace size for all three heights and methods.

	Method and Brace Size								
Story	**ELF**	**ARM**	**DIST**	**ELF**	**ARM**	**DIST**	**ELF**	**ARM**	**DIST**
18	17	17	17						
17	17	12	14						
16	17	15	14						
15	16	12	14						
14	16	14	14						
13	16	11	11						
12	16	11	11	15	15	15			
11	16	11	11	15	10	12			
10	15	11	11	15	13	12			
9	15	11	11	15	10	12	14	14	14
8	14	11	11	14	12	12	14	9	11
7	13	11	11	14	10	10	14	12	11
6	12	11	11	13	10	10	13	9	11
5	11	11	11	11	10	10	12	11	11
4	9	11	11	10	10	10	11	9	9
3	7	10	10	8	9	9	9	8	8
2	5	9	9	6	8	8	7	7	7
1	3	6	6	3	5	5	4	5	5
Total	**235**	**205**	**208**	**139**	**122**	**125**	**98**	**84**	**87**

5 ANALYSIS UNDER SUITES

In order to analyze these distributions a time history analysis must be run for each distribution under the same ground motions and scaled appropriately. For each distribution at the three building heights the designs were subjected to time history analysis under two suites of 20 ground motion records each. These two suites shall be referred to as Long and Short. Long is applicable to buildings with a long building period and Short to buildings with a short building period.

The building model for these tests was altered from the original optimization model in order to coincide with parallel research. The story heights remained the same while the bay widths were increased to 30 feet. Instead of a single brace configuration a chevron configuration with two braces in each bay was used. Another bay was added increasing the building footprint and making a total of five bays on each face of the structure. The maximum displacements for each story were recorded for each ground motion in the suite. The median and 85th percentile was then calculated for each level under all of the ground motion records in the suite. The range of displacements was also calculated for each distribution at each building height.

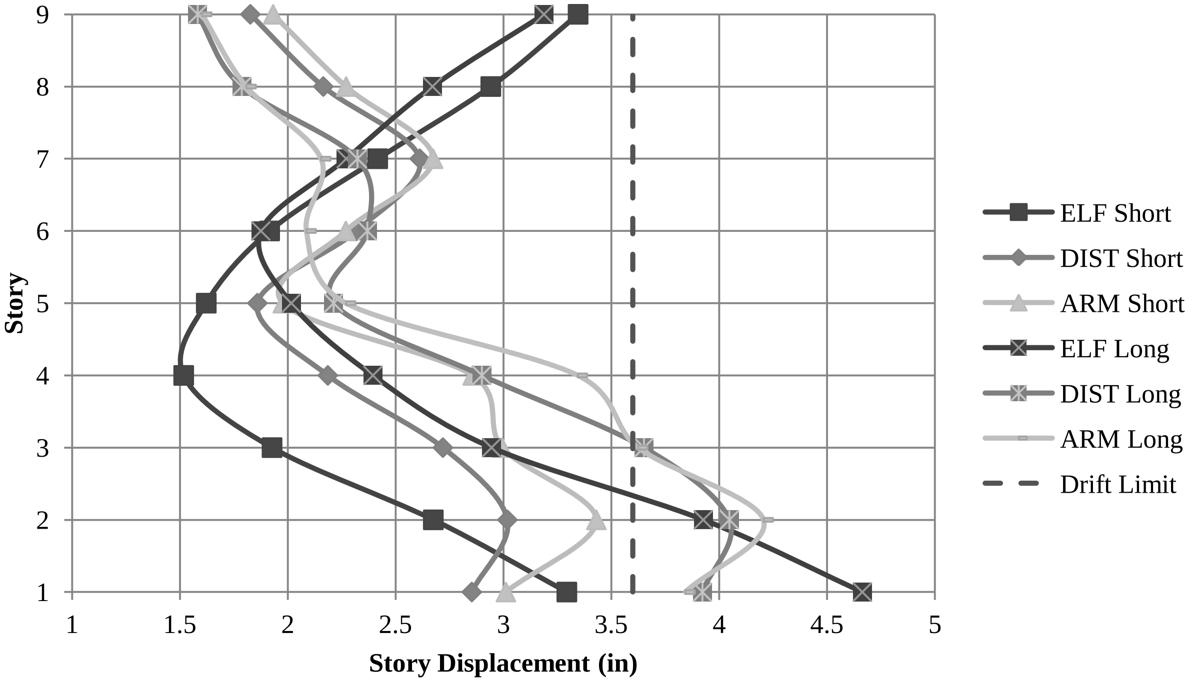

Figure 5-1: Story displacements for each distribution under both suites for 9-stories.

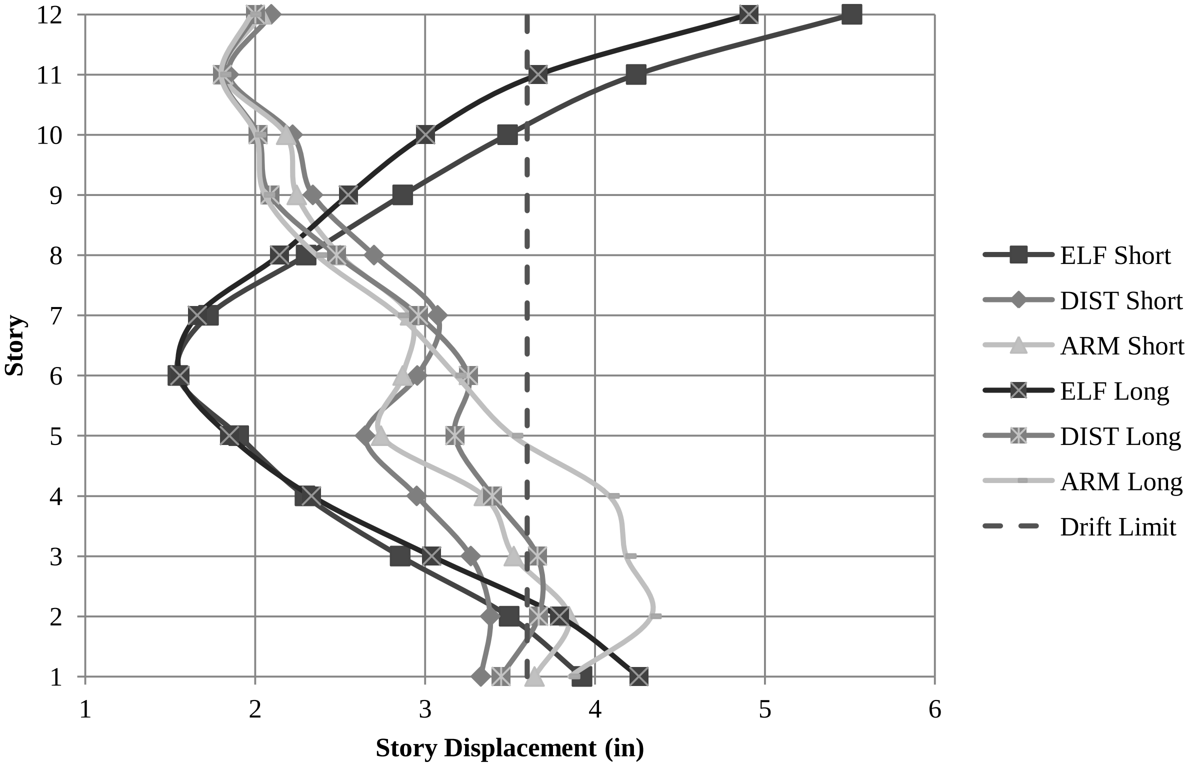

Figure 5-2: Story displacements for each distribution under both suites for 12-stories.

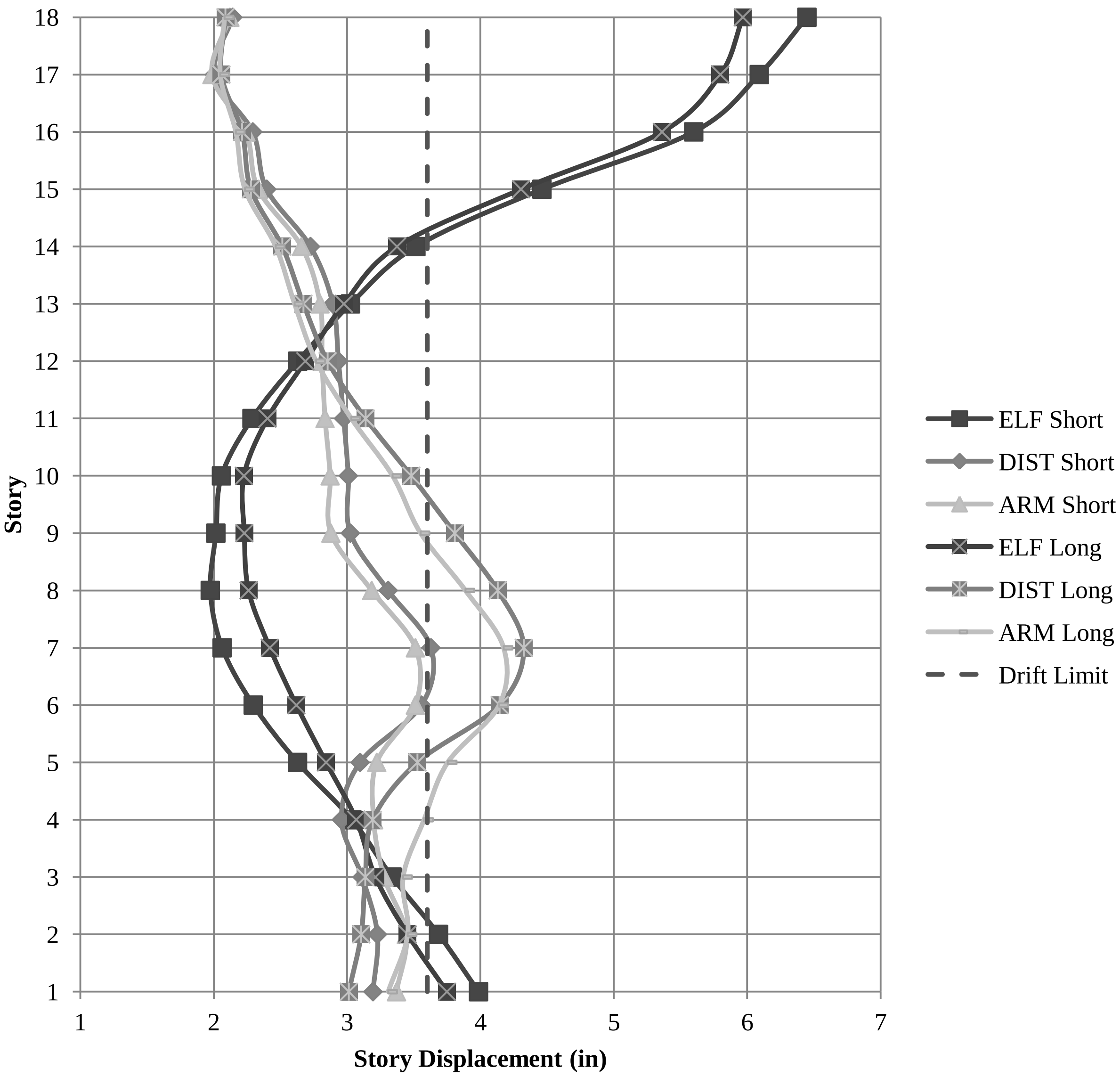

Figure 5-3: Story displacements for each distribution under both suites for 18-stories.

Table 5-1: Maximum, minimum and spread displacements for 9-stories.

	9 Stories		
	Min	**Max**	**Spread**
ELF Short	1.52	3.35	1.83
DIST Short	1.83	3.02	1.19
ARM Short	1.93	3.43	1.50
ELF Long	1.88	4.66	2.79
Dist Long	1.58	4.05	2.46
ARM Long	1.60	4.21	2.60

Table 5-2: Maximum, minimum and spread displacements for 12-stories.

12 Stories			
	Min	**Max**	**Spread**
ELF Short	1.54	5.51	3.97
DIST Short	1.84	3.38	1.54
ARM Short	1.81	3.85	2.04
ELF Long	1.56	4.91	3.35
Dist Long	1.81	3.67	1.86
ARM Long	1.80	4.33	2.53

Table 5-3: Maximum, minimum and spread displacements for 18-stories.

18 Stories			
	Min	**Max**	**Spread**
ELF Short	1.97	6.45	4.47
DIST Short	2.01	3.63	1.62
ARM Short	1.99	3.51	1.53
ELF Long	2.23	5.97	3.74
Dist Long	2.06	4.32	2.27
ARM Long	2.05	4.17	2.13

6 DISCUSSION OF RESULTS

Using a genetic algorithm within an optimization program, optimum designs were obtained for a 9-, 12- and 18- story BRBF where the key design variables were total brace area and column demands. These optimum designs were studied and two brace distributions were formulated. These distributions were analyzed using a nonlinear time history procedure under two suites of 20 earthquakes each and performance was compared to a design obtained from the ELF procedure.

An interesting result from this study was that the two distributions that coincided with the optimums found from the genetic algorithm yielded better overall performance as it relates to story drifts while also substantially lowering the total brace area over the entirety of the building. As the building height increased the difference in displacement spread between the ELF and DIST/ARM distributions increased. The effectiveness of the DIST and ARM distributions in lowering the story displacements became more prevalent as the building height increased. Either distribution would be a better method than the ELF method when wanting to reduce story drifts.

Balling, Balling and Richards (2009) found for shorter buildings that the optimum brace distribution was linear. As the building height increases the optimum brace distributions loses its linearity in the majority of the stories while the upper most stories continue the linear trend. This could be a result from higher mode shapes playing a larger role in the response of the building to the ground motions.

6.1 **Possible Future Work**

This researched focused on a typical structure and primarily the two distributions developed. The following is a list of areas that can be considered beyond the basic outline presented in this study and areas that require further research.

1. Consider other distributions that perform better under the suites of earthquakes.
2. Develop the nonlinear time history analysis code to record maximum column demands and observe the behavior of the frames designed by the ELF method compared to other distributions.
3. Look into the dynamic response of the building using the equation of motion in order to explain why the optimum designs contain certain properties as the building height increases.
4. Perform physical testing in order to verify that the distributions discovered in this research do perform as predicted.

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APPENDIX A. SOURCE CODE

The following pages contain a typical input file, template file and main program file for the non linear time history analysis program for a suite of earthquakes.

A.1 Input ARM Distribution for 9-Stories

```
#This is an input file that defines parameters used to generate modes.
#The same input file can be used in conjunction with static.tcl,
#dynamic.tcl, and pushover.tcl to perform different kinds of analyses
#on the same systems.
#

#Files that will be referenced
source Template42/template42.tcl
source Template12/template12.tcl
source OtherProcedures/generalProcedures.tcl
source OtherProcedures/combinedTemplates.tcl
source OtherProcedures/postProcessing.tcl
source OtherProcedures/rotationSpringParameters.tcl
source OtherProcedures/LookupShapeProp.tcl
```

```
#variables that specify file locations

set filename "OtherProcedures/AISC_Database.csv"

set GMdir "GMfiles"

set dataDir "Results"


#General variables that typically apply to multiple templates

set numFrames 1

set numStories 9


set storyHeights [list [expr 15*12] [expr 15*12] [expr 15*12]\

                                          [expr 15*12] [expr 15*12] [expr 15*12]\

                                          [expr 15*12] [expr 15*12] [expr 15*12]]

set bayWidth [expr 30*12]

set Es 29000

set Fy 50

set columnTransfCode 1; #0 is linear, 1 is P-Delta, 2 is corotational

set beamTransfCode 1; #0 is linear, 1 is P-Delta, 2 is corotational


#All of the templates used to define the system

set diffTemplates [list 42 12]
```

```
#Loads for P-Delta effects
set colLoad -521; #should be -1022 for Daniels study.
set colPointLoads [list $colLoad $colLoad $colLoad $colLoad $colLoad $colLoad $colLoad $colLoad $colLoad]


#Specific properties for template 41 (p-delta columns)
#set pDeltacolAreas [list 1000 1000 1000 1000 1000 1000]; #add an entry for every story
#set pDeltacolIs [list 1 1 1 1 1 1]; #add an entry for every story
#set pDeltacolZx [list 448 448 448 196 196 196]; #add an entry for every story
set storyHeight [expr 15.0*12]
set pDeltacolAreas [list 272 272 272 272 168 168 140 140 99]; #add an entry for every story
set pDeltacolIs [list 7273 7273 7273 7273 3602 3602 3602 3602 1874]; #add an entry for every story
set pDeltacolZx [list 9000 9000 9000 9000 9000 9000 9000 9000 9000]; #add an entry for every story
set pDeltacolumnProps [list $pDeltacolAreas $pDeltacolIs $pDeltacolZx]
set pDeltacolumnFixity 0; #0 is pinned at base, 1 is fixed


#Specifiy properties for template 12 (single bay chevron BRBF)
```

```
        set numFrames12 1
        set baseFixity12 0; #0 pinned, 1 fixed
        set beamFixity12 0; #0 pinned, 1 fixed
        set colShapes [list W14X311 W14X311 W14X211 W14X211 W14X132 W14X132
W14X68 W14X68 W14X48];
                                #these are the columns sizes at each story.  Use upper-case X.\
                                There should be        (number of stories) columns in the list.
        set columnProps12 [LookupShapeProp $filename $colShapes];

        set beamShapes [list W14X48 W14X48 W14X48 W14X48 W14X48 W14X48 W14X48
W14X48 W14X48];
                                #These are beam sizes at each story
        set beamProps12 [LookupShapeProp $filename $beamShapes]

        set brAreas12 [list 14 9 12 9 11 9 8 7 5]; #this is the primary design
        set brIs12 [list 300 300 300 300 300 300 300 300 300]; #not used
        set brZx12 [list 300 300 300 300 300 300 300 300 300]; #not used
        set braceProps12 [list $brAreas12 $brIs12 $brZx12]

        #brace parameters
        set Fyb 38
        set b_Brace 0.003
        set R0_Brace 20;
```

```
set cR1_Brace 0.925; # parameter for the change of R with cyclic loading history
set cR2_Brace 0.15; # parameter for the change of R with cyclic loading history
set stiffnessFactor 1.6;  #accounts for nonuniform core area
set a1 0.065
set a2 1
set a3 0.045
set a4 1



#Defining the masses (not required for pushover or static analyses)
set sameStoryMass 1.532;
set storyMasses [list $sameStoryMass $sameStoryMass $sameStoryMass\
                                                            $sameStoryMass     $sameStoryMass
$sameStoryMass $sameStoryMass $sameStoryMass $sameStoryMass]
set templateForMasses 42

#Defining the template used for the drift recorders
set driftTemplate 42
```

A.2 Template

```
# ---------------------------------------------------------------------------
#
# template41nodesAndElements
```

```
#
#               Template 41 is for a P-Delta column.
#               Template 41 is a simpler version of template 42 that does not include
#               yielding springs (just elastic beam-column elements)
#               Determines the coordinates for all the nodes for the 41 template.
#               Uses OpenSees node command to define them.
#               Defines beam elements
#
#
# Arguments:
#numStories     the number of stories in the frame
#storyHeights   list will all of the story heights (in inches)
#Es     modulus of elasticity for steel
#Fy     yield strength of beams and columns
#columnProps            a list with lists of column properties
#columnTransfCode       indicates the type of geometric transformation\
for     the column elements(0 linear, 1 pdelta)
#fixity         0=pinned, 1=fixed at the base; should be pinned if being
#used as a p-delta column; may be fixed to do simple
#system studies.
#
# Results:
#Sends out commands to define the node coordinates and\
```

```
elements in OpenSees
#
#
# Author:
#Paul Richards, adapted from template42 on 8/6/13
#
proc template42nodesAndElements {numStories storyHeights\
	Es Fy columnProps\
	columnTransfCode\
	fixity} {

	set template 42

#########################################################################
	#preparatory stuff so that the beam and column elements can be defined
	set columnAreas [lindex $columnProps 0]
	set columnIs [lindex $columnProps 1]

	#################################################################
	#info for geometric transformation, same for all procedures
	set columnTransfTag [expr $template*100 + 0]; #0 is for columns
	if {$columnTransfCode==0} {
	geomTransf Linear $columnTransfTag;
```

```
} elseif {$columnTransfCode==1} {
geomTransf PDelta $columnTransfTag;
} else {
geomTransf Corotational $columnTransfTag;
}

################################################################################
#need to generate the locations for the x-grids and y-grids
set ygridElevations 0
set cumElevation 0
foreach height $storyHeights {
set cumElevation [expr $cumElevation + $height]
lappend ygridElevations $cumElevation
}

set frameNum 1
#Define the base nodes in the 0 plane and fix those at y-grid 0
set ygrid 0
set Y [lindex $ygridElevations $ygrid];
set X 0
set xgrid 0
set plane 0
set nodeID [expr $template*100000000 + $frameNum*1000000 \
```

```
+ $plane*10000 + $ygrid*100 + $xgrid]
node $nodeID $X $Y;
fix $nodeID 1 1 1;
#Define the column nodes (2 plane) and elements (1 plane)
for {set ygrid 0} {$ygrid < $numStories} {incr ygrid} {
set xgrid 0
set plane 2
set nodeI [expr $template*100000000 + $frameNum*1000000\
+ $plane*10000 + $ygrid*100 + $xgrid]
#this node only needs to be defined when ygrid is 0\
#for other cases it is defined by the j-node
if {$ygrid == 0} {
set X 0
set Y [lindex $ygridElevations $ygrid];
node $nodeI $X $Y;
}
set nodeJ [expr $template*100000000 + $frameNum*1000000 \
+ $plane*10000 + [expr $ygrid + 1]*100 + $xgrid]
set X 0
set Y [lindex $ygridElevations [expr $ygrid + 1]];
node $nodeJ $X $Y;
set plane 1
set elemID [expr $template*100000000 + $frameNum*1000000 \
```

```
	+ $plane*10000 + [expr $ygrid + 1]*100 + $xgrid]

	set area [lindex $columnAreas [expr $ygrid]]

	set I [lindex $columnIs [expr $ygrid]]

	element elasticBeamColumn $elemID $nodeI $nodeJ $area $Es\

	$I $columnTransfTag

	}

	#Constrain the bottom column nodes to the base nodes.

	set ygrid 0

	set xgrid 0

	set plane 2

	set nodeSlave [expr $template*100000000 + $frameNum*1000000\

	+ $plane*10000 + $ygrid*100 + $xgrid]

	set plane 0

	set nodeMaster [expr $template*100000000 + $frameNum*1000000\

	+ $plane*10000 + $ygrid*100 + $xgrid]

	if {$fixity==0} {

	equalDOF $nodeMaster $nodeSlave 1 2

	} else {

	equalDOF $nodeMaster $nodeSlave 1 2 3

	}

	puts "node constraints defined"

puts "nodes and elements defined"

}
```

```
# ---------------------------------------------------------------------------

#

# template42gravity

#

#	Applies gravity loads to the P-Delta column.

#	Different point loads may be applied at each level.

#	Uses OpenSees pattern command to define them.

#	Performs gravity analysis.

#

# Arguments:

#	numStories	the number of stories in the frame

#	colPointLoads a list of the point loads to be placed at each level

#

# Results:

#	Defines and applies gravity loads

#

#

# Author:

#	Paul Richards, modified from template42 on 8/6/13

#

# ---------------------------------------------------------------------------


proc template42gravity {numStories colPointLoads} {
```

```
set template 42

#Define the load pattern
pattern Plain 100 Constant {
set frameNum 1
set plane 2; #top of column node plane
for {set ygrid 1} {$ygrid <= $numStories} {incr ygrid} {
set xgrid 0
set nodeID [expr $template*100000000 + $frameNum*1000000 \
+ $plane*10000 + $ygrid*100 + $xgrid]
load $nodeID 0.0 [lindex $colPointLoads [expr $ygrid -1 ]] 0.0
puts "load $nodeID 0.0 [lindex $colPointLoads [expr $ygrid -1 ]] 0.0"
}

}

#Perform the static gravity analysis
set Tol 1.0e-6;            # convergence tolerance for test
variable constraintsTypeGravity Plain;          # default
constraints $constraintsTypeGravity; # how it handles boundary\
conditions
numberer RCM;          # renumber dof's to minimize\
```

```
band-width (optimization),\
if you want to
system BandGeneral;          # how to store and solve the \
system of equations in the\
analysis (large model:\
try UmfPack)
test NormDispIncr $Tol 6;    # determine if convergence has\
been achieved at the end of\
an iteration step
algorithm Newton;            # use Newton's solution algorithm:\
updates tangent stiffness at\
every iteration
set NstepGravity 1;                    # apply gravity in 10 steps
set DGravity [expr 1.0/$NstepGravity];         # load increment
integrator LoadControl $DGravity;              # determine the next time step\
for an analysis
analysis Static;             # define type of analysis static\
or transient



analyze $NstepGravity;                                   # apply gravity
loadConst -time 0.0;                                     # maintain constant gravity\
loads and reset time to zero
```

```
puts "gravity applied and analyzed"
}
```

A.3 Main Program

```
#***********************************************************************
#                         dynamic.tcl                          *
#***********************************************************************
# Description: This script is executed in OpenSees by typing "source
#	dynamic.tcl" on the OpenSees command line), this script will create
#	models based on templates and analyze the models under combined gravity
#	and earthquake loading.
#	Many comments are included since it is intended that this will be the
#	users first experience with the TCL programming language and with OpenSees.
#
# Overview: This script is divided into 9 parts.
#	1. Reading in the source files for dynamic analysis procedures
#	2. Defining the parameters for the dynamic analysis
#	3. Building the P-Delta column (represents the "non-seismic" parts of bldg)
#	4. Building the seismic frames using various templates
#	5. Assigning masses and performing eigenvalue analysis
#	6. Defining damping
#	7. Defining recorders to get output
#	8. Running the dynamic analyses (with gravity loads applied)
```

```
#	9. Post-processing results



#**********************************************************************
# 1. Reading in the source files for inputs and dynamic analysis procedures   *
#**********************************************************************
# Description: OpenSees consists of a series of pre-defined procedures that
#	are used to build models and subject them to earthquake loads.
#	In addition to these "built-in" procedures, custom procedures
#	can be defined to assist in the definition of models and processing
#	results.  The commands below read in files that define custom procedures.
#	For information on the procedures defined by a file, open the
#	particular file in a text editor and read the comments.

source example18.tcl
source OtherProcedures/dynamicAnalysisProcedures.tcl
source OtherProcedures/combinedTemplates.tcl
source OtherProcedures/eqSuites.tcl

#source OtherProcedures/rotationalSprings.tcl

#**********************************************************************
# 2. Defining the parameters for the dynamic analysis			*
```

```
#*************************************************************************************
# Description: The variables defined below control features of the dynamic
#         analyses.

set GMdir "GMfiles";             #Folder with the Ground Motion files
set suiteName "LongPeriod"; #Name of the earthquake suite to be used for
#analyses.  Possible suites for use are defined
#in Other Procedures/eqSuites.tcl
set suiteList [defineSuite $suiteName];  #procedure to populate the list of GMs

set Sds 1.40;                    #Parameter to define a design spectra that can be
                                 #used to scale Ground Motion files
set Sd1 0.8;                     #Another parameter to define a design spectra
set tLong 8;                     #Another parameter to define a design spectra

set periodList1 [createList 0.01 2 0.01]
set periodList2 [createList 2.1 6 0.1]
set periodList3 [createList 7.0 20 1]
set periodList [concat $periodList1 $periodList2 $periodList3]
                #The Period lists above are for scaling ground motions.

set GMdirection 1;               #Direction of ground acceleration, 1 means x-direction
set IDLoadTagDyn 20;             #Dynamic analysis needs a unique load tag. The actual
```

```
        #value is arbitrary.  It just has to be different from
        #the gravity load tag.
set IDLoadTagGravity 100;   #See comment above.
set dtAnalysis 0.005;  #Time step used for the analysis.  Does not have to
                #match the time step for the earthquake records

set dampingZeta 0.02;#Level of Rayleigh Damping assinged to the model
set extraTime 60;               #This is extra time added to dynamic analysis after the
                #groud acceleration stops to allow the building to stop
                #vibrating so residual deformations can be measured.

#**********************************************************************
# 3. Building the P-Delta column                                      *
#**********************************************************************
# Description: The P-Delta column represents the parts of the building that
#       are not the seismic frames.  It is helpful to always have it defined.  If
#       it turns out that the P-Delta is not needed, or if results need to be
#       checked without its influence, very small properties can be assigned
#       for the column cross-sections and no gravity loads can be assigned (these
#       things are defined in the input.tcl file).  For the dynamic analysis,
#       masses will be assigned to the P-Delta column, and drift recorders at each
#       level will be used to determine story drifts.
```

```
wipe all;                                   # clear memory of past model definitions

model BasicBuilder -ndm 2 -ndf 3;   # ndm = #dimension, ndf = #dofs


template42nodesAndElements $numStories $storyHeights\

                                    $Es $Fy        $pDeltacolumnProps\

                                    $columnTransfCode\

                                    $pDeltacolumnFixity


#*********************************************************************

# 4. Building the seismic frames using various templates              *

#*********************************************************************

# Description: The seismic frames are defined using templates.  The input.tcl

#       file must source the template procudures you are going to use, and must

#       provide all of the input parameters required for the templates (see

#       comments in the input.tcl file).

#       Any number of templates may be used.


template12 $numFrames12 $numStories $storyHeights $bayWidth $baseFixity12\

                $beamFixity12 $Es\

                $columnProps12 $beamProps12 $braceProps12\

                $columnTransfCode $beamTransfCode\

                $Fyb $b_Brace $R0_Brace $cR1_Brace $cR2_Brace $stiffnessFactor\

                $a1 $a2 $a3 $a4
```

```
combineTemplates $numStories $diffTemplates


#********************************************************************
# 5. Assigning masses and performing eigenvalue analysis                    *
#********************************************************************
# Description: The masses are assigned at each level of the P-Delta column.
#	eigenvalue analysis determines the natural period of the frames.


assignMasses $templateForMasses $numFrames $numStories $storyMasses
set naturalPeriods [periods $numStories]
set firstModePeriod [lindex $naturalPeriods 0]


#*******************************************************************
# 6. Defining damping                                                   *
#********************************************************************
# Description: Raleigh damping is defined based on zeta in the first and third
#	modes


defineDamping $dampingZeta $naturalPeriods


#********************************************************************
```

```
# 7. Initialize some of the results lists                                    *
#********************************************************************
# Description: These lists will be populated at the system is analyzed under
#          each earthquake in the suite.  They need to be initialized to start
set storyNumbers [list]
for {set i 1} {$i <= $numStories} {incr i} {
          lappend storyNumbers $i
          }
set cumStoryDriftList $storyNumbers
set cumResidualDriftList $storyNumbers


#********************************************************************
# 8. Running the dynamic analyses (with gravity loads applied)                *
#********************************************************************
# Description: This part of the script executes a loop that analyzes the system
#          under each of the earthquakes in the suite.  The basic steps for each
#          earthquake are: a) determine the appropriate scale factor for the
#          b) clear all previous analysis results and recorders, d) define recorders,
#          e) apply the gravity loads, f) read in the earthquake record, g) do the
#          dynamic analysis under the earthquake, h) remove results from OpenSees
#          memory (they are recorded in the output files),  i) do some post-processing
#          to identify maximum responses from the particular earthquake, prior to the
#          output files being overwritten by the next earthquake.  These steps are
```

```
#	repeated for every earthquake record in the suite.

foreach GM $suiteList {

		# (a)
		set earthquakeSpectra [readEarthquakeSpectraCustom $GM]

		set period [lindex $earthquakeSpectra 0]
		set acc [lindex $earthquakeSpectra 1]


		# (b)
		set scaleFactor [computeScalingFactor $earthquakeSpectra $firstModePeriod $Sd1 $Sds $tLong ]
		#set scaleFactor [expr 1.5*$scaleFactor]; #uncomment for MCE
		puts "scaleFactor is: $scaleFactor"

		# (c)
		reset;
		wipeAnalysis;
		remove recorders;

		# (d)
```

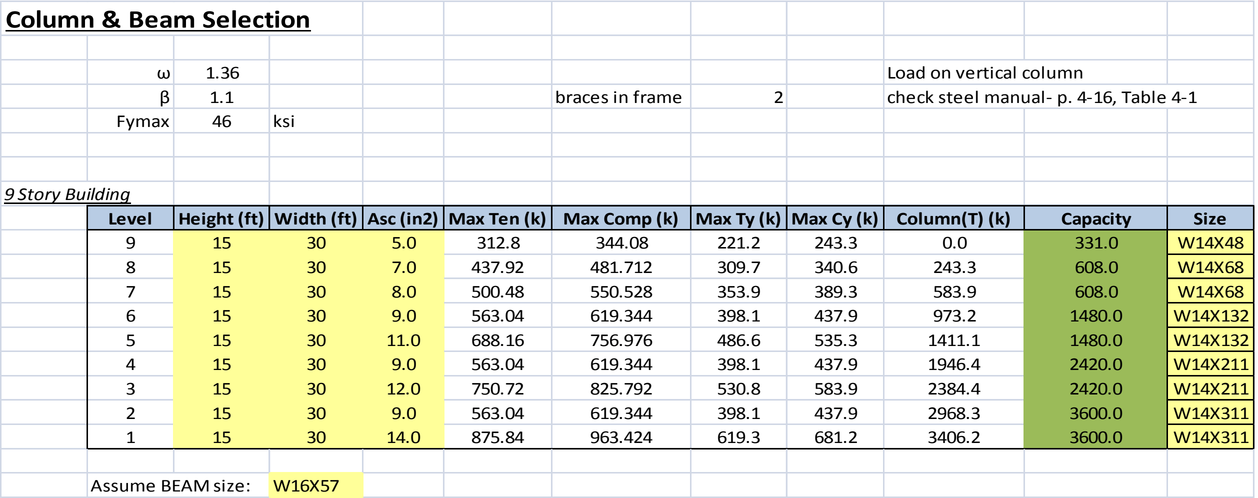
```
setupStoryDriftRecorders $dataDir $numStories $driftTemplate;
#template32recorders $numFrames $numStories
#template02recorders $numFrames $numStories $dataDir
template12recorders $numFrames $numStories $dataDir

# (e)
template42gravity $numStories $colPointLoads

# (f)
set inFilename "GMfiles/$GM.AT2"
set outFilename "GMfiles/$GM.dat"
ReadRecord $inFilename $outFilename dt nPts
puts "dt is $dt"
puts "nPts is $nPts"

# (g)
source OtherProcedures/dynamicAnalysisSettings.tcl
runDynamicAnalysis $scaleFactor $GM $GMdirection $IDLoadTagDyn\
                $nPts $dtAnalysis $dt $IDLoadTagGravity $extraTime


# (h)
wipeAnalysis;
```

```
            remove recorders;

            # (i)
            set driftSummaries [findMaxFromOut $numStories $dataDir storydrift]
            set storyDrifts [lindex $driftSummaries 0]
            set residualDrifts [lindex $driftSummaries 1]
            puts "story drift is: $storyDrifts"
            puts "residual drifts are: $residualDrifts"

    set cumStoryDriftList [cumStoryDrifts $numStories $storyDrifts  $cumStoryDriftList]
    set cumResidualDriftList [cumStoryDrifts $numStories $residualDrifts $cumResidualDriftList]
            puts "cumStoryDriftList: $cumStoryDriftList"
            puts "cumResidualDriftList: $cumResidualDriftList"

    }

    #*********************************************************************
    # 9. Post-processing results
    #*********************************************************************
    # Description: After the system has been analyzed under each earthquake, and
    #       maximum responses have been identified, median and 85th percentile values
    #       can be calculated.  At the end, the wipe all command removes the model from
```

```
    #    memory.

        set avgStoryDriftList [avgStoryDrifts $cumStoryDriftList $numStories]
        puts "average story drift demands $avgStoryDriftList"

        set avgResidualDriftList [avgStoryDrifts $cumResidualDriftList $numStories]
        puts "average residaul drift demands $avgResidualDriftList"

        set avgPlusStoryDriftList [avgPlusStoryDrifts $cumStoryDriftList $numStories]
        puts "average plus story drift demands $avgPlusStoryDriftList"

        set avgPlusResidualDriftList [avgPlusStoryDrifts $cumResidualDriftList $numStories]
        puts "average plus residual drift demands $avgPlusResidualDriftList"

    wipe all;
```

APPENDIX B. FRAME CALCULATIONS

B.1 Column Design for ARM Distribution 9-Stories



Column & Beam Selection

ω 1.36

β 1.1

Fymax 46 ksi

braces in frame 2

Load on vertical column

check steel manual- p. 4-16, Table 4-1

9 Story Building

Level	Height (ft)	Width (ft)	Asc (in2)	Max Ten (k)	Max Comp (k)	Max Ty (k)	Max Cy (k)	Column(T) (k)	Capacity	Size
9	15	30	5.0	312.8	344.08	221.2	243.3	0.0	331.0	W14X48
8	15	30	7.0	437.92	481.712	309.7	340.6	243.3	608.0	W14X68
7	15	30	8.0	500.48	550.528	353.9	389.3	583.9	608.0	W14X68
6	15	30	9.0	563.04	619.344	398.1	437.9	973.2	1480.0	W14X132
5	15	30	11.0	688.16	756.976	486.6	535.3	1411.1	1480.0	W14X132
4	15	30	9.0	563.04	619.344	398.1	437.9	1946.4	2420.0	W14X211
3	15	30	12.0	750.72	825.792	530.8	583.9	2384.4	2420.0	W14X211
2	15	30	9.0	563.04	619.344	398.1	437.9	2968.3	3600.0	W14X311
1	15	30	14.0	875.84	963.424	619.3	681.2	3406.2	3600.0	W14X311

Assume BEAM size: W16X57

B.2 Building and P-Delta Column Design for 9-Stories

$S_{DS} := 1.4$ $S_{D1} := 0.8$ $L_{bay} := 30\text{ft}$ $n_{bay} := 5$ $h := 15\text{ft}$ $n := 9$ $N_x := 4$ $N_y := 2$

$R := 8$ $I_e := 1$ $h_n := h \cdot \frac{n}{\text{ft}} = 135$ $h_{parapet} := 42\text{in}$ $L_{overhang} := 2\text{ft}$ $C_d := 5$

$w_{floor} := 100\text{psf}$ $w_{roof} := 100\text{psf}$ $w_{seismic} := 90\text{psf}$ $w_{extwall} := 25\text{psf}$ $w_{live} := 50\text{psf}$

ASCE 7-10 parameters

Finding Cs

$C_t := 0.03$ $x := 0.75$ $C_u := 1.4$

$T_a := C_t \cdot {h_n}^{x} = 1$ $T := C_u \cdot T_a = 2$

$$C_{s1} := \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} = 0 \qquad C_{s2a} := \frac{S_{D1}}{T \cdot \left(\frac{R}{I_e}\right)} = 0 \qquad C_{s2b} := 0.044 \cdot S_{DS} \cdot I_e = 0$$

Cs will be the smaller of Cs1, Cs2a.
If Cs2a is smaller, it should not be smaller than Cs2b.

$C_s := C_{s2a} = 0$

Finding Area

$length := n_{bay} \cdot L_{bay} + 2 \cdot L_{overhang} = 154\,\text{ft}$

$width := n_{bay} \cdot L_{bay} + 2 \cdot L_{overhang} = 154\,\text{ft}$

$A := length \cdot width = 23716\,\text{ft}^2$

Live Load Reduction

$A_t := L_{bay} \cdot L_{bay} = 900\,\text{ft}^2$

$$LL_{red1} := w_{live} \cdot \left(0.25 + \frac{15}{\sqrt{4 \cdot \frac{A_t}{\text{ft}^2}}}\right) = 25 \cdot \text{psf} \qquad LL_{red2} := 0.4 \cdot w_{live} = 20 \cdot \text{psf}$$

If LLred1 is less than LLred2, use LLred2. Going to assume all maxed out.

$w_{redlive} := LL_{red2} = 20 \cdot \text{psf}$

Wall and Parapet Weight for Whole Building

$w_{wall} := w_{extwall} \cdot (h) \cdot (2length + 2width) = 231 \cdot \text{kip}$

$w_{parapet} := w_{extwall} \cdot h_{parapet} \cdot (2 \cdot length + 2 \cdot width) = 54 \cdot \text{kip}$

Weight Calculation

$w_f := w_{seismic}$ $w_r := w_{seismic}$

$$W_f := (w_f) \cdot A + w_{wall} = 2365.4 \cdot kip$$

$$W_r := (w_r) \cdot A + 0.5 w_{wall} + w_{parapet} = 2303.8 \cdot kip$$

$$W := (n - 1) \cdot W_f + W_r = 21227 \cdot kip$$

$$\frac{W_f}{g \cdot 4} = 1.532 \cdot \frac{kip \cdot sec^2}{in}$$

Total Base Shear

$$V := C_s \cdot W = 1276 \cdot kip$$

Vertical Distribution of Base Shear

$$h_1 := \frac{h_n}{(n)} \quad h_2 := \frac{2 \cdot h_n}{n} \quad h_3 := \frac{3 \cdot h_n}{n} \quad h_4 := \frac{4 \cdot h_n}{n} \quad h_5 := \frac{5 \cdot h_n}{n} \quad h_6 := \frac{6h_n}{(n)}$$

$$w_1 := W_f \quad w_2 := W_f \quad w_3 := W_f \quad w_4 := W_f \quad w_5 := W_f \quad w_6 := W_f$$

$$h_7 := \frac{7h_n}{(n)} \quad h_8 := \frac{8h_n}{(n)} \quad h_9 := \frac{9h_n}{(n)}$$

$$w_7 := W_f \quad w_8 := W_f \quad w_9 := W_r$$

If T < 0.5, k=1
If T > 2.5, k=2
Otherwise,

$T = 2$

$k := .5 \cdot T + .75 = 2$

$$whsum := w_1 \cdot h_1{}^k + w_2 \cdot h_2{}^k + w_3 \cdot h_3{}^k + w_4 \cdot h_4{}^k + w_5 \cdot h_5{}^k + w_6 \cdot h_6{}^k + w_7 \cdot h_7{}^k + w_8 \cdot h_8{}^k + w_9 \cdot h_9{}^k$$

$$F_1 := \frac{w_1 \cdot h_1{}^k}{whsum} \cdot V = 10 \cdot kip \quad F_2 := \frac{w_2 \cdot h_2{}^k}{whsum} \cdot V = 30 \cdot kip \quad F_3 := \frac{w_3 \cdot h_3{}^k}{whsum} \cdot V = 56 \cdot kip$$

$$F_4 := \frac{w_4 \cdot h_4{}^k}{whsum} \cdot V = 89 \cdot kip \quad F_5 := \frac{w_5 \cdot h_5{}^k}{whsum} \cdot V = 127 \cdot kip \quad F_6 := \frac{w_6 \cdot h_6{}^k}{whsum} \cdot V = 169 \cdot kip$$

$$F_7 := \frac{w_7 \cdot h_7{}^k}{whsum} \cdot V = 216 \cdot kip \quad F_8 := \frac{w_8 \cdot h_8{}^k}{whsum} \cdot V = 267 \cdot kip \quad F_9 := \frac{w_9 \cdot h_9{}^k}{whsum} \cdot V = 313 \cdot kip$$

Check:

$$F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8 + F_9 = 1276 \cdot kip \qquad V = 1276 \cdot kip$$

Centers of Rigidity and Mass

Since the building and brace configuration is symmetric and square, the centers of mass and rigidity will act at the same point. Considering accidental torsion, the center of mass will be displaced 5% of the building length away from the center of rigidity. Again, since the building and braces are symmetric and square, the maximum force in any brace will be the same.

$x_r := \frac{\text{length}}{2} = 77\,\text{ft}$ from left $\qquad y_r := \frac{\text{length}}{2} = 77\,\text{ft}$ from bottom

$x_m := x_r + 0.05\cdot(\text{length}) = 85\,\text{ft}$ from left $\qquad y_m := y_r = 77\,\text{ft}$ from bottom

Spring distances from Center of Mass

$d_n := y_r \qquad d_s := y_r \qquad d_w := x_m = 85\,\text{ft} \qquad d_e := \text{length} - x_m = 69\,\text{ft}$

Maximum Axial Brace Force at each story

Roof

$F_9 = 312.8\cdot\text{kip} \quad M_9 := F_9\cdot(x_m - x_r) = 2408\,\text{ft}\cdot\text{kip}$

$$F_{9f} := \frac{F_9}{N_y} = 156.4\cdot\text{kip}$$

$$F_{9m} := \frac{M_9\cdot d_e}{2\cdot d_n{}^2 + 2\cdot d_s{}^2 + 2\cdot d_w{}^2 + 2\cdot d_e{}^2} = 3.5\cdot\text{kip}$$

$$F_{9max} := F_{9f} + F_{9m} = 159.9\cdot\text{kip}$$

8th floor

$F_8 = 266.5\cdot\text{kip} \quad M_8 := F_8\cdot(x_m - x_r) = 2052\,\text{ft}\cdot\text{kip}$

$$F_{8f} := \frac{F_8}{N_y} = 133.3\cdot\text{kip}$$

$$F_{8m} := \frac{M_8\cdot d_e}{2\cdot d_n{}^2 + 2\cdot d_s{}^2 + 2\cdot d_w{}^2 + 2\cdot d_e{}^2} = 3\cdot\text{kip}$$

$$F_{8max} := F_{8f} + F_{8m} = 136.3\cdot\text{kip}$$

7th floor

$F_7 = 215.8\cdot\text{kip} \quad M_7 := F_7\cdot(x_m - x_r) = 1662\,\text{ft}\cdot\text{kip}$

$$F_{7f} := \frac{F_7}{N_y} = 107.9\cdot\text{kip}$$

$$F_{7m} := \frac{M_7\cdot d_e}{2\cdot d_n{}^2 + 2\cdot d_s{}^2 + 2\cdot d_w{}^2 + 2\cdot d_e{}^2} = 2.4\cdot\text{kip}$$

$$F_{7max} := F_{7f} + F_{7m} = 110.3\cdot\text{kip}$$

6th floor

$F_6 = 169.1 \cdot kip \quad M_6 := F_6 \cdot (x_m - x_r) = 1302\, ft \cdot kip$

$$F_{6f} := \frac{F_6}{N_y} = 84.5 \cdot kip$$

$$F_{6m} := \frac{M_6 \cdot d_e}{2 \cdot {d_n}^2 + 2 \cdot {d_s}^2 + 2 \cdot {d_w}^2 + 2 \cdot {d_e}^2} = 1.9 \cdot kip$$

$$F_{6max} := F_{6f} + F_{6m} = 86.4 \cdot kip$$

5th floor

$F_5 = 126.7 \cdot kip \quad M_5 := F_5 \cdot (x_m - x_r) = 976\, ft \cdot kip$

$$F_{5f} := \frac{F_5}{N_y} = 63.4 \cdot kip$$

$$F_{5m} := \frac{M_5 \cdot d_e}{2 \cdot {d_n}^2 + 2 \cdot {d_s}^2 + 2 \cdot {d_w}^2 + 2 \cdot {d_e}^2} = 1.4 \cdot kip$$

$$F_{5max} := F_{5f} + F_{5m} = 64.8 \cdot kip$$

4th floor

$F_4 = 89 \cdot kip \qquad M_4 := F_4 \cdot (x_m - x_r) = 686\, ft \cdot kip$

$$F_{4f} := \frac{F_4}{N_y} = 44.5 \cdot kip$$

$$F_{4m} := \frac{M_4 \cdot d_e}{2 \cdot {d_n}^2 + 2 \cdot {d_s}^2 + 2 \cdot {d_w}^2 + 2 \cdot {d_e}^2} = 1 \cdot kip$$

$$F_{4max} := F_{4f} + F_{4m} = 45.5 \cdot kip$$

3rd floor

$F_3 = 56.5 \cdot kip \quad M_3 := F_3 \cdot (x_m - x_r) = 435\, ft \cdot kip$

$$F_{3f} := \frac{F_3}{N_y} = 28.2 \cdot kip$$

$$F_{3m} := \frac{M_3 \cdot d_e}{2 \cdot {d_n}^2 + 2 \cdot {d_s}^2 + 2 \cdot {d_w}^2 + 2 \cdot {d_e}^2} = 0.6 \cdot kip$$

$$F_{3max} := F_{3f} + F_{3m} = 28.9 \cdot kip$$

2nd floor

$$F_2 = 29.7 \cdot \text{kip} \qquad M_2 := F_2 \cdot (x_m - x_r) = 229\,\text{ft} \cdot \text{kip}$$

$$F_{2f} := \frac{F_2}{N_y} = 14.9 \cdot \text{kip}$$

$$F_{2m} := \frac{M_2 \cdot d_e}{2 \cdot d_n^{\,2} + 2 \cdot d_s^{\,2} + 2 \cdot d_w^{\,2} + 2 \cdot d_e^{\,2}} = 0.3 \cdot \text{kip}$$

$$F_{2max} := F_{2f} + F_{2m} = 15.2 \cdot \text{kip}$$

1st floor

$$F_1 = 9.9 \cdot \text{kip} \qquad M_1 := F_1 \cdot (x_m - x_r) = 77\,\text{ft} \cdot \text{kip}$$

$$F_{1f} := \frac{F_1}{N_y} = 5 \cdot \text{kip}$$

$$F_{1m} := \frac{M_1 \cdot d_e}{2 \cdot d_n^{\,2} + 2 \cdot d_s^{\,2} + 2 \cdot d_w^{\,2} + 2 \cdot d_e^{\,2}} = 0.1 \cdot \text{kip}$$

$$F_{1max} := F_{1f} + F_{1m} = 5.1 \cdot \text{kip}$$

Axial Force in Each Brace

$$L_{brace} := \sqrt{\left(\frac{L_{bay}}{2}\right)^2 + h^2} = 21\,\text{ft} \qquad x_{dir} := \frac{0.5 \cdot L_{bay}}{L_{brace}} = 1 \qquad F_y := 38\text{ksi}$$

$$P_9 := \frac{F_{9max}}{2x_{dir}} = 113 \cdot \text{kip}$$

$$P_8 := \frac{F_{8max} + F_{9max}}{2x_{dir}} = 209 \cdot \text{kip}$$

$$P_7 := \frac{F_{9max} + F_{8max} + F_{7max}}{2x_{dir}} = 287 \cdot \text{kip}$$

$$P_6 := \frac{F_{9max} + F_{8max} + F_{7max} + F_{6max}}{2x_{dir}} = 349 \cdot \text{kip}$$

$$P_5 := \frac{F_{9max} + F_{8max} + F_{7max} + F_{6max} + F_{5max}}{2x_{dir}} = 394 \cdot \text{kip}$$

$$P_4 := \frac{F_{9max} + F_{8max} + F_{7max} + F_{6max} + F_{5max} + F_{4max}}{2x_{dir}} = 427 \cdot \text{kip}$$

$$P_3 := \frac{F_{9max} + F_{8max} + F_{7max} + F_{6max} + F_{5max} + F_{4max} + F_{3max}}{2x_{dir}} = 447 \cdot kip$$

$$P_2 := \frac{F_{9max} + F_{8max} + F_{7max} + F_{6max} + F_{5max} + F_{4max} + F_{3max} + F_{2max}}{2x_{dir}} = 458 \cdot kip$$

$$P_1 := \frac{F_{9max} + F_{8max} + F_{7max} + F_{6max} + F_{5max} + F_{4max} + F_{3max} + F_{2max} + F_{1max}}{2x_{dir}} = 461 \cdot kip$$

Area for BRBF

$$A_1 := \frac{P_1}{0.9 \cdot F_y} = 13.49 \cdot in^2 \qquad A_4 := \frac{P_4}{0.9 \cdot F_y} = 12.47 \cdot in^2 \qquad A_7 := \frac{P_7}{0.9 \cdot F_y} = 8.4 \cdot in^2$$

$$A_2 := \frac{P_2}{0.9 \cdot F_y} = 13.38 \cdot in^2 \qquad A_5 := \frac{P_5}{0.9 \cdot F_y} = 11.53 \cdot in^2 \qquad A_8 := \frac{P_8}{0.9 \cdot F_y} = 6.12 \cdot in^2$$

$$A_3 := \frac{P_3}{0.9 \cdot F_y} = 13.07 \cdot in^2 \qquad A_6 := \frac{P_6}{0.9 \cdot F_y} = 10.19 \cdot in^2 \qquad A_9 := \frac{P_9}{0.9 \cdot F_y} = 3.31 \cdot in^2$$

Typical Gravity Column Design

$$P_{gcol} := \left(1.2w_{floor} + 1.6 \cdot w_{redlive}\right) \cdot A_t = 137 \cdot kip \qquad KL_y := 15ft$$

Column Demands

$$P_{gc1} := n \cdot P_{gcol} = 1231 \cdot kip$$

$$P_{gc2} := (n - 1) \cdot P_{gcol} = 1094 \cdot kip$$

$$P_{gc3} := (n - 2) \cdot P_{gcol} = 958 \cdot kip$$

$$P_{gc4} := (n - 3) \cdot P_{gcol} = 821 \cdot kip$$

$$P_{gc5} := (n - 4) \cdot P_{gcol} = 684 \cdot kip$$

$$P_{gc6} := (n - 5) \cdot P_{gcol} = 547 \cdot kip$$

$$P_{gc7} := (n - 6) \cdot P_{gcol} = 410 \cdot kip$$

$$P_{gc8} := (n - 7) \cdot P_{gcol} = 274 \cdot kip$$

$$P_{gc9} := (n - 8) \cdot P_{gcol} = 137 \cdot kip$$

Shapes chosen from AISC Manual Table 4-1

Stories 1 and 2: W14X132 $\phi P_{n1} := 1480kip$

Stories 3 and 4: W14X132 $\phi P_{n3} := 1480kip$

Stories 5 and 6: W14X82 $\phi P_{n5} := 735kip$

Stories 7 and 8: W14X68 $\phi P_{n7} := 608kip$

Story 9: W14X48 $\phi P_{n9} := 331kip$

Demands on P-Delta Column

$$P_{pdeltaf} := 1.2 \cdot w_{floor} \cdot \frac{A}{N_x} + 1.2 \frac{w_{wall}}{N_x} + 1.6 \cdot w_{redlive} \cdot \frac{A}{N_x} = 971 \cdot kip$$

$$P_{pdeltar} := 1.2 \cdot w_{roof} \cdot \frac{A}{N_x} + 1.2 \frac{0.5w_{wall} + w_{parapet}}{N_x} + 1.6 \cdot w_{redlive} \cdot \frac{A}{N_x} = 952 \cdot kip$$

$P_{pdelta} := P_{pdeltaf} + P_{pdeltar} = 1923 \cdot kip \quad n_x := 3.5 \quad n_y := 3.5$

P-Delta Column Inputs

Stories 1 and 2 W14X132

$I_{x1} := 1530in^4 \quad I_{y1} := 548in^4 \quad A_{mem1} := 38.8in^2$

$I_1 := n_x \cdot I_{x1} + n_y \cdot I_{y1} = 7273 \cdot in^4 \quad I_2 := I_1$

$A_1 := (n_x + n_y) \cdot A_{mem1} = 272 \cdot in^2 \quad A_2 := A_1$

Stories 3 and 4 W14X132

$I_{x3} := 1530in^4 \quad I_{y3} := 548in^4 \quad A_{mem3} := 38.8in^2$

$I_3 := n_x \cdot I_{x3} + n_y \cdot I_{y3} = 7273 \cdot in^4 \quad I_4 := I_3$

$A_3 := (n_x + n_y) \cdot A_{mem3} = 272 \cdot in^2 \quad A_4 := A_3$

Stories 5 and 6 W14X82

$I_{x5} := 881in^4 \quad I_{y5} := 148in^4 \quad A_{mem5} := 24.0in^2$

$I_5 := n_x \cdot I_{x5} + n_y \cdot I_{y5} = 3602 \cdot in^4 \quad I_6 := I_5$

$A_5 := (n_x + n_y) \cdot A_{mem5} = 168 \cdot in^2 \quad A_6 := A_5$

Stories 7 and 8 W14X68

$I_{x7} := 722in^4 \quad I_{y7} := 121in^4 \quad A_{mem7} := 20in^2$

$I_7 := n_x \cdot I_{x5} + n_y \cdot I_{y5} = 3602 \cdot in^4 \quad I_8 := I_7$

$A_7 := (n_x + n_y) \cdot A_{mem7} = 140 \cdot in^2 \quad A_8 := A_7$

Story 9 W14X48

$I_{x9} := 484in^4 \quad I_{y9} := 51.4in^4 \quad A_{mem9} := 14.1in^2$

$I_9 := n_x \cdot I_{x9} + n_y \cdot I_{y9} = 1874 \cdot in^4 \quad I_{10} := I_9$

$A_9 := (n_x + n_y) \cdot A_{mem9} = 99 \cdot in^2 \quad A_{10} := A_9$

P-Delta Summary

$P_{pdelta} = 1923 \cdot kip$

$A_1 = 272 \cdot in^2$

$A_2 = 272 \cdot in^2$

$A_3 = 272 \cdot in^2$

$A_4 = 272 \cdot in^2$

$A_5 = 168 \cdot in^2$

$A_6 = 168 \cdot in^2$

$A_7 = 140 \cdot in^2$

$A_8 = 140 \cdot in^2$

$A_9 = 99 \cdot in^2$

$I_1 = 7273 \cdot in^4$

$I_2 = 7273 \cdot in^4$

$I_3 = 7273 \cdot in^4$

$I_4 = 7273 \cdot in^4$

$I_5 = 3602 \cdot in^4$

$I_6 = 3602 \cdot in^4$

$I_7 = 3602 \cdot in^4$

$I_8 = 3602 \cdot in^4$

$I_9 = 1874 \cdot in^4$