DEPARTMENT OF CIVIL ENGINEERING
BRIGHAM YOUNG UNIVERSITY

INVESTIGATION OF REINFORCED CONCRETE COLUMNS SUBJECTED TO BIAXIAL BENDING IN TENSION ZONE

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This project, by Suresh Sabhlok, is accepted in its present form by the Department of Civil Engineering Science of Brigham Young University as satisfying in part the requirements for the Degree of Master of Civil Engineering.

Arnold Wilson  
Chairman, Advisory Committee

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December 14, 1972  
Date

James L. Barton  
Chairman, Major Department
Dedicated To My
Loving Parents
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CHAPTER I

OBJECT

The purpose of this project was to show how a column subject to biaxial bending and in the tensile zone is penalized very much when the equation:

$$\frac{M_x}{M_{ox}} + \frac{M_y}{M_{oy}} \leq 1$$

where $M_{ox} = 0.4$ As $f_y (d - ds)$ is used when $M_y$ tends to zero, and if the above equation is used to compare the case where the equation is not used and bending about one axis is applied.
CHAPTER 2

INTRODUCTION TO THE PROBLEM

General Theory of Biaxial Bending

The problem of how to design reinforced concrete columns which are subjected to combined action of axial load and bending moment has attracted considerable attention for more than half a century. It was of frequent interest to practicing engineers of civil and structural calling in the past; it is safe to bet it will be in the foreseeable future.

Years ago reinforced columns were quite generally designed as axially loaded without length effect except in extreme cases. To make such practice safe, factors of safety were kept extremely high. Next, the importance of end moment was partially recognized. When ultimate strength design was introduced in the Code* in 1956, it included mandatory design for at least a minimum eccentricity of 0.5t on spiral column and 0.10t on tied columns.

The design of columns under axial load accompanied by bending moments about principal axes is complicated. The ultimate load for specific values of eccentricities is influenced by such factors as dimensions of cross section, percentage reinforcement, number and arrange-

*American Concrete Institute Building Code
ment of bars, yield stress of steel, strength of concrete and depth of cover. Furthermore, the nonrectangular shape of the compression zone (because of the variation of the neutral axis in depth and direction and with eccentricities and the nonlinear stress distribution in concrete) introduced additional complications. Even for the simpler case of an eccentrically loaded column, use of available formulas is restricted to a particular disposition of steel, i.e. all steel being concentrated in opposite faces. If the bars are distributed among all faces, the ultimate load can be determined only by process of trial and error.

The methods available for design of biaxially loaded columns are: (1) trial and error procedure and (2) determination of ultimate loads from failure surface in column.

Recently published methods are based on the concept of failure surface in columns shown, i.e. fig. 1 and fig. 2.
Notable among them are contributions of Pannell and Bresler. Pannell has shown that the equivalent uniaxial moment $M_{uxo}$ of the radial moment $M_u$ corresponding to any ultimate load $P_u$ can be determined with the aid of parameters $N$, the deviation factor and $\theta$, the ratio of $M_{ux}/M_{uy}$. The theoretical load corresponding to the calculated uniaxial moment is then determined from the major axis interaction diagram.

This procedure determined the load from the moments and is likely to give rise to possible errors in the estimation of ultimate load, especially when the failure is controlled by the tension and the calculated equivalent uniaxial moment is nearly equal in magnitude to the balanced failure value. The method necessitates careful determination of moment in such cases, for, as seen from the interaction diagram, the load falls rapidly for little change in moment at one set of tension failure conditions.

Of the two methods proposed by Bresler, the equation:

$$\frac{1}{P_i} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} - \frac{1}{P_o}$$

in Method A is simple and easy to apply. The equation, though exact for material obeying Hook's law, gives surprisingly satisfactory results when applied to concrete. As for the equation:

$$\left(\frac{M_{ux}}{M_{uxo}}\right)^r + \left(\frac{M_{uy}}{M_{uyo}}\right)^r = 1$$
in Method B it has been shown that no single value can be assigned to the exponent \( r \) to represent the true shape of load contour in all cases. Even for the simpler case of an eccentrically loaded column, use of available formulas is restricted to a particular disposition of steel, i.e. all steel being concentrated in opposite faces. If the bars are distributed among all faces, the ultimate load can be determined only by process of trial and error.

The methods available for design of biaxially loaded columns are (1) trial and error procedure and (2) determination of ultimate loads from failure surface in column.

Recently published methods are based on the concept of failure surface in columns shown in fig. 1 and fig. 2
Now in ACI Design Handbook of Reinforced Concrete it has been shown how to determine percentage of steel in the following cases: (1) axial load and bending about one axis and (2) axis load and bending about two axes.

According to the book, the typical axial load versus moment interaction diagram has been divided into three regions for design purposes. Region I and II are in the compression control range while Region III is in tension control range. Columns with small eccentricities designated in the Handbook less than $e_a/t$ are designed directly by equation (14-1) of ACI 318-63 and the moment is disregarded in accordance with Chapter 14 of the book. Region II corresponds to the sloping portion of the interaction diagram in compression control range as defined by equation (14-9) of ACI 318-63. It is possible to convert the moment to an equivalent axial load and the column can again be designated by equation (14-1). Column in the tension control (Range III) can be designed by converting the effect of the applied axial load and the moment to an equivalent pure moment, Moe.

When designing a column subjected to moments about two directions, the method outlined for columns with moment in one direction must be adjusted as follows:

$$\frac{e_{xt}}{b} + \frac{e_{yt}}{t} \leq 1$$
for biaxial bending in the tension control range, it is not possible to derive equivalent pure moments, Moe and therefore a different approach must be used. Equation 14-14 in ACI Code enables the designer to compare the applied moments $M_x$ and $M_y$ with the allowable moments $M_{ox}$ and $M_{oy}$ corresponding to axial load $N = 0$. If the applied load is greater than zero and less than $P_b$, the designer may use a more exact method of comparing applied moments to allowable moments. The moment capacities $M_{xx}$ and $M_{yy}$ of a given column section subjected to an axial load $N$ are obtained from:

$$M_{xx} = N \left( \frac{d'_{x} b}{12} \right) + \left( \frac{p'_{x} t b^2}{c'_{x}} \right)$$

$$M_{yy} = N \left( \frac{d'_{y} t}{12} \right) + \left( \frac{p'_{y} b t^2}{c'_{y}} \right)$$

If these moment capacities and the applied moments $M_x$ and $M_y$ satisfy the equation:

$$\frac{M_x}{M_{xx}} + \frac{M_y}{M_{yy}} \leq 1 \quad (14 - 14)$$

then the section meets the design requirement.

From the investigator's viewpoint the following questions were raised:

1. Is it possible to use equation (14 - 14) when moment about one axis is known and moment about the other axis is zero?
2. Is the same amount of steel required when the procedure
given in ACI handbook for bending about one axis is applied
and equation (14-14) is used, when $M_y$ tends to zero.

3. Is the equation (14-14) the exact equation for theoretical
and practical purposes?

Thus the investigation is done to show how the column is penalized
when subjected to biaxial bending and in the tensile zone when the
equation $\frac{M_x}{M_{ox}} + \frac{M_y}{M_{oy}} \leq 1$ is used.
CHAPTER 3

INVESTIGATION

The investigation described here is carried out to seek answers to the above questions.

Symbols and Notations

- \( A_g \) - Gross area of concrete section
- \( A_s \) - Area of tensile reinforcement of beams or columns
- \( A_{st} \) - Total area of longitudinal reinforcement in columns
- \( b \) - Width of the column
- \( C' \) - \( \frac{12}{0.17 m^2 f'c} \) design coefficient used for rectangular tied columns in Region III
- \( e \) - Eccentricity measured from tensile steel axis (in.)
- \( e_a \) - Maximum permissible eccentricity of Pa
- \( e_b \) - Maximum permissible eccentricity of Pb
- \( E \) - Eccentricity measured from tensile steel axis (ft.)
- \( E_c \) - Modulus of elasticity of concrete
- \( E_s \) - Modulus of elasticity of steel
- \( f_a \) - Axial load divided by area of column \( A_g \)
- \( f'c \) - Ultimate compressive strength of concrete
- \( f_s \) - Stress in column reinforcement
- \( f_y \) - Yield strength of reinforcement
- \( F_a \) - \( 0.34 (1 + p_m) f'c \), used in column design
g - ratio of diameter of circle (gt) through bar center-lines, or distance (gt) between bars at opposite faces of column to overall dimension (t)

h - actual unsupported length of column

h' - effective length of column

I - moment of inertia

K - \( \frac{As^2}{As^1} \) for column design using reinforcement on four faces

m - \( \frac{f_y}{0.85f_c} \)

M - external moment (ft. kips or in. kips)

Mr - resisting moment of concrete stresses

M_b - moment corresponding to Pb, in column design

M_o - allowable moment when column section is in pure flexure

M_{oe} - equivalent pure moment used for columns in Region III, tension controls

M_s - F_b S, in column design

M' - \( \frac{M}{f_c A_g} \), abscissa of column interaction diagrams

N - external force or load (kips)

N' - \( \frac{N}{f_c A_g} \)

p - ratio of area of tnesile reinforcement to effective area of concrete in column

p_g - ratio of area of vertical reinforcement to gross area, A_g

p_1 - ratio of reinforcement (A_{s1}) to gross area (A_g)

p_2 - ratio of reinforcement (A_{s2}) to gross area (A_g)

P - axial load capacities

P' - \( \frac{P}{f_c A_g} \), ordinate of column interaction diagrams
CHAPTER 4

SOLVED PROBLEMS

Six examples are solved to investigate the problem.

Example #1

Consider a column having moment about one axis only and $M_y$ tends to zero.

\[ f_y = 40,000 \text{ psi} \]
\[ f_s = 16,000 \text{ psi} \]
\[ b = 12'' \]
\[ t = 12'' \]
\[ g = 0.6 \]
\[ M = 19.0 \text{ kft} \]
\[ N = 55.5 \text{ k} \]

from appendix area of steel require for this data is equal to 2.59 sq. in. Let us see if the column is safe when subjected to biaxial bending and with amount of steel equal to 2.59 sq. in.

Data:

\[ f_y = 40,000 \text{ psi} \]
\[ f_s = 16,000 \text{ psi} \]
\[ f_c = 4,000 \text{ psi} \]
\[ b = 12'' \]
\[ t = 12'' \]
\[ g = 0.6 \]
\[ M_x = 19.0 \text{ kft} \]
\[ M_y = 12.12 \text{ kft} \]
\[ N = 55.5 \text{ k} \]

**Solution**

If we use the same area of steel \( A_s = 2.59 \text{ sq. in.} \)

\[
\frac{M_x}{M_{ox}} + \frac{M_y}{M_{oy}} \leq 1
\]

\[
M_{ox} = M_{oy} = 0.40 A_{sfy} (d-d')
\]

\[
= 0.40 \times \frac{2.59}{2} \times 40 (10-2)
\]

\[
= 166
\]

\[
\frac{M_x}{M_{ox}} + \frac{M_y}{M_{oy}}
\]

\[
= \frac{19}{166} (12) + \frac{12.12}{166} (12) = 2.25
\]

This means the column will fail with this amount of steel. Now in the problem, even if \( M_y \) tends 0

\[
\frac{M_x}{M_{ox}} = 1.38 > 1 \quad \text{Hence still the column will fail.}
\]

As we can see from the above problem that even the design is safe when solved by unaxial bending, the steel is insufficient when solved by expression for biaxial bending even when \( M_y = 0 \).
Consider example problem No. II.

Data:
\[ f_y = 60,000 \]
\[ f'_c = 4,000 \]
\[ b = 18" \]
\[ t = 22" \]
\[ g = 0.8 \]
\[ N = 158 \text{ K} \]
\[ M_x = 236 \text{ Kft.} \]

From appendix area of steel requires = 11.53 sq. in.

Example Problem #2

Now consider the same example with eccentricity in two axes and let us check if it is safe with \( A_s = 11.53 \) sq. in.

Data
\[ f_y = 60,000 \text{ psi} \]
\[ f'_c = 4,000 \text{ psi} \]
\[ n = 8 \]
\[ b = 12" \]
\[ t = 22" \]
\[ g_y = 0.8 \]
\[ g_x = 0.7 \]
\[ M_x = 236 \text{ kft.} \]
\[ M_y = 98 \text{ kft.} \]
\[ N = 158 \text{ k.} \]
Compute \[
\frac{N}{f'_{CAG}} = \frac{158}{4 \times \frac{18}{22}} = 0.10
\]

From table 26 for \(g_y = 0.7\) and \(g_x = 0.8\)

Minimum value of \[
\frac{P_b}{f'_{CAG}} = 0.14
\]

so tension controls the design.

Use same amount of steel as used in previous problem.

\[
A_{st} = 14 \text{ sq. in.}
\]

\[
P_g = \frac{14}{396} = 0.0354
\]

Properties of reinforcement about \(y-y\) axis

\[
A_{s1} = 2 \times 4 \times 1.0 = 8 \text{ sq. in.}
\]

\[
A_{s2} = 2 \times 3 \times 1.0 = 6 \text{ sq. in.}
\]

\[
P_{y1} = \frac{8}{396} = 0.0202
\]

\[
P_{y2} = \frac{6}{396} = 0.0152
\]

\[
P'_y = P_{y1} + 0.5 P_{y2}
\]

\[
= 0.0202 + 0.5 \times 0.0152 = 0.0278
\]

Properties of reinforcement about \(x-x\) axis:

\[
A_{s1} = 2 \times 5 \times 1.0 = 10 \text{ sq. in.}
\]

\[
A_{s2} = 2 \times 2 \times 1.0 = 4 \text{ sq. in.}
\]

\[
P_{y1} = \frac{10}{396} = 0.0253
\]

\[
P_{y2} = \frac{4}{396} = 0.0101
\]

\[
P'_y = P_{y1} + 0.5 P_{y2}
\]
\[= 0.0253 + 0.5 \times 0.0101 = 0.0303\]

From table 34 obtain \(D'\) value for \(P_g\)

\[0.035 \quad \text{Now} \quad g_y = 0.7\]

\[k = \frac{A_{s2}}{A_{s1}} = \frac{6}{8} = 0.75\]

\[D'_y = 0.130\]

for \(g_x = 0.8 \quad k = \frac{A_{s2}}{A_{s1}} = \frac{4}{10} = 0.4\)

\[D'_x = 0.113\]

from table 26 \(f_y = 60,000 \quad f'_c = 4,000\)

\[g_y = 0.7 \quad c'_y = 1.43\]

\[g_x = 0.8 \quad c'_x = 1.25\]

\[M_y = N \left( \frac{D'_y t}{12} \right) + p'_y \left( \frac{bt^2}{py} \right)\]

\[= 158 \left( \frac{0.130 \times 2}{12} \right) + 0.0303 \left( \frac{18 \times 22 \times 22}{1.43} \right)\]

\[= 37.6 + 187.0 = 224.6\]

Similarly \(M_{xx}\) is found as follows:

\[M_{xx} = N \left( \frac{D'_x b}{12} \right) + p'_x \left( \frac{tb^2}{cx} \right)\]

\[= 158 \left( \frac{113 \times 18}{12} \right) + 0.0272 \left( \frac{22 \times 18^2}{1-25} \right)\]

\[= 269 + 14.20 = 283.20\]
Solving equation (14 - 14) of ACI (318-63)

\[
\frac{M_x}{M_{xx}} + \frac{M_y}{M_{yy}} = \frac{236}{2245} + \frac{98}{283.20}
\]

\[
= 1.04 + 0.35 = 1.39 \geq 1
\]

Hence, even the design is safe when solved by uniaxial bending the steel is not sufficient when checked by equation (14 - 14).

**Problem #3**

Let us solve for one more column having axial load 225 K and moment of 225 K.ft. and about 2 percent of hard grade steel.

Calculate eccentricity \( e = \frac{M}{N} \)

\[
= \frac{225 \times 1000}{225,000} \times 12 = 12''
\]

Estimate preliminary cross section. Assume \( e/t = 0.80 \)

Trial equivalent axial load \( P \)

\[
P = N (1 + B \frac{e}{t})
\]

\[
= 225,000 (1 + 3 \times .80) = 765,000#
\]

for \( P = 0.02 \)

\[
P = 0.85 (0.25 \times 5000 + 20,000 \times 0.02) A_g
\]

\[
765,000 = 1400 A_g
\]

\[
A_g = 546 \text{ in.} \quad p = 23.4 \times 23.4
\]

Say = 22" x 22"
To determine area of reinforcement:

equivalent load = 765,000#

value of concrete = 0.85 x 5000 x .25 x 22^2 = 515,000#

value of steel = 0.85 x 20,000 A_s = 250,000#

Use 12 - #10 A_s = 14.7 sq. in.

\[
\frac{P_g A_s}{A_g} = \frac{15.24}{222} \quad e_a = (0.67 p_g M \quad 0.17) d
\]

\[
e_a = 0.0316 \quad = (0.67 \times .0316) (11.75) + .17 \quad 19.5
\]

\[
\frac{M f_y}{.85 f_c} = \frac{5000}{.84 (5000)} \quad e_b = 8.18" \quad 12"
\]

\[
e_b = 11.75
\]

So tension controls.

When tension controls, design is according to section 1407 (C). The allowable moment varies linearly from M_o to M_b. As N varies from zero to N_b, N_b can be computed from equation (14-9) by substituting N_b for M_b.

Determine f_a substituting N_b for N

\[
f_a = \frac{N}{A_g} = \frac{N_b}{A_g} = \frac{N_b}{(22^2) (22)} = 0.00206 \times N_b
\]

Calculate F_a using equation (14-10)

\[
F_a = 0.34 (1 + p_g M) f_c
\]
\[ = 0.34 \left(1 + 0.0316 \times 11.75\right) 5000 \]
\[ = 2330 \text{ psi} \]

Calculate \( F_b = 0.45 \quad f_c' = 0.45 \times 5000 = 2250 \text{ psi} \)

\[
\text{Iconc.} = \frac{bd^3}{12} = \frac{22 \times 22^3}{12} = 19,500 \text{ in.}^4
\]

\[
I_{\text{steel}} = 15.24 \left(2 \times 7.1 - 1\right) 8.5^2 = \frac{14,500 \text{ in.}^4}{34,000 \text{ in.}^4}
\]

\[
S = \frac{34,000}{11.0} = 3,090 \text{ in.}^3
\]

Determine \( f_b \)

\[
f_b = \frac{M_c}{I} = \frac{M}{S} = \frac{N_b \times 8.18}{3090}
\]

\[ = 0.00265 N_b \]

Solve equation (14.9)

\[
\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1
\]

\[
\frac{0.00206 N_b}{2230} + \frac{0.00265 N_b}{2250} = 1
\]

Solving \( N_b = 486,000 \text{#} \)

\[
M_b = N_b e_b = 486 \times \frac{8.18}{12} = 331 \text{ kft.}
\]

Allowable \( M_0 = 0.40 A_{sfy}(d - d') \)

\[
M_0 = 0.40 \times \frac{15.24}{2} \times 50 \left(19.5 - 2.5\right)
\]

\[ = 216 \text{ kft.} \]
Allowable moment on column by linear proportion:

\[ M = 216 + \frac{225}{486} (331-216) \]

269 kft. \( \geq \) 225 kft.

Problem #3A

Now consider problem with biaxial bending

Data:

- \( N = 225 \text{ K} \)
- \( M_x = 225 \text{ kft.} \)
- \( M_y = 75 \text{ kft.} \)
- \( f_c' = 5000 \text{ kft.} \)
- \( f_y = 50,000 \)
- \( b = 22 \)
- \( t = 22" \)
- \( g_y = 0.8 \)
- \( g_x = 0.7 \)

Solution:

\[ \text{Ast. 12 bars of #10} \]
\[ \text{Ast} = \frac{12 \times 1.27}{1.27} = 15.24 \]

\[ p_g = 15.24 = 0.0315 \]

Properties of reinforcement about y-y axis:

\[ A_{s1} = 2 \times 1.27 \times 1 = 10.10 \text{ sq. in.} \]
\[ A_{s2} = 2 \times 2 \times 1.27 = 5.08 \text{ sq. in.} \]

\[ p_{x1} = \frac{10.16}{22 \times 22} = 0.021 \]

\[ p_{x2} = \frac{5.08}{22 \times 22} = 0.0105 \]

\[ p_x = 0.021 + 0.25 \times 0.01 = 0.023. \]

From table 34 obtain \( D' \) value

for \( p_g = 0.0315 \)

\[ g_y = 0.8 \]

\[ k = \frac{A_{s2}}{A_{s1}} = \frac{4}{8} = 0.5 \]

read \( D'_y = 0.127 \)

for \( g_x = 0.7 \)

\[ k = 0.5 \]

\[ D'_x = 0.140 \]

From table 26 for \( f_y = 50,000 \)

\[ f'_c = 5000 \]

\[ g_y = 0.8 \]

\[ c'_y = 1.50 \]

\[ g_x = 0.7 \]

\[ c'_x = 1.71 \]
\[ M_{yy} = N \frac{D_{yt}}{12} + p_y \frac{bt^2}{c_y} \]
\[ = 225 \frac{0.127 \times 22}{12} + 0.023 \frac{22 \times 22^2}{1.50} \]
\[ = 225 (0.233) + 0.023 (710) \]
\[ = 52.5 + 71.0 \]
\[ = 123.5 \]

\[ M_{xx} = N \frac{D_{xb}}{12} + p_x \frac{tb^2}{c_x} \]
\[ = 225 \frac{0.14 \times 22}{12} + 0.023 \frac{22 \times 22^2}{1.71} \]
\[ = 225 (.26) + .023 (6250) \]
\[ = 58.5 + 145.0 = 203.5 \]

\[ \frac{M_x}{M_{xx}} + \frac{M_y}{M_{yy}} = \frac{225}{203.5} + \frac{.75}{123.5} = \]

\[ 1.11 + 0.60 = 1.71 \geq 1 \text{ Unsafe} \]

Thus it is clear from above example that even \( M_y \) tends to 0 the value of equation = 1.11 and still the column will fail. From all above solved problems, we conclude the following results.
CHAPTER 5

CONCLUSIONS

From the six solved problems on unaxial bending and biaxial bending, the following conclusions have been drawn:

1. Column design in tensile zone is penalized very much when the equation
\[
\frac{M_x}{M_{ox}} + \frac{M_y}{M_{oy}} \leq 1
\]
is used.

2. When the column in the tensile zone is designed having moment about the axes, equation (14 - 14) applies if and only if \(N = 0\).

3. The equation \(\frac{M_x}{M_{ox}} + \frac{M_y}{M_{oy}}\) seems to be conservative. Thus from the above conclusions following recommendations for future programs are suggested.
CHAPTER 6

RECOMMENDATIONS

As observed from the six solved problems the equation
\[
\frac{M_x}{M_{ox}} + \frac{M_y}{M_{oy}}
\]
is very conservative and hence further investigation

on this above equation should be done or in other words the equation

(14 - 14) should be modified. Testing various columns in the tension
zone in future programs may help to revise the equation
\[
\frac{M_x}{M_{ox}} + \frac{M_y}{M_{oy}} \leq 1
\]

It is also recommended that if the column subjected to biaxial
bending is designed by working stress design, it should be checked by
ultimate stress design.
BIBLIOGRAPHY


APPENDIX

Problem #1

Solution of Problem 1 and 2.

Data

\[ f_y = 40,000 \text{ psi} \]
\[ f_s = 16,000 \text{ psi} \]
\[ b = 12'' \]
\[ t = 12'' \]
\[ g = 0.6 \]
\[ M = 19.0 \text{ ft. k.} \]
\[ N = 55.5 \text{ k.} \]

Solution

Following the ACI procedure for uniaxial bending

Compute \[ \frac{N}{f_c A_g} = \frac{55.5}{40 \times 12 \times 12} = 0.096. \] From Table 26,

for \( g = 0.6 \): \[ \frac{P_b}{f_c A_g} \] varies from 0.20 to 0.19. Since \[ \frac{N}{f_c A_g} \]

\[ \frac{P_b}{f_c A_g} \] Hence tension controls and column is designed in Region III.

First trial assume \( M_{oe} = M_x \) to get approximate value of \( P_g \) from
Table 26. $c' = 2.0$

Compute $A_{st} = \frac{c'}{t} \cdot \text{Moe}$

$$= \frac{2.50}{12} \times 19.00 = 3.96 \text{ sq. in.}$$

$$p_g = \frac{A_{st}}{A_g} = \frac{3.96}{12 \times 12} = 0.027$$

Second trial: Assume $p_g = 0.027$ from table, $D' = 0.14$

$c' = 2.50$

$
M_{oe} = M_x - D' \frac{N t}{12}
$

$$= 19.0 - 0.14 \frac{55.5 \times 12}{12}$$

$$= 11.25$$

$A_{st} = \frac{c'}{t} \cdot M_{oe}$

$$= \frac{2.50 \times 11.25}{12} = 2.34 \text{ sq. in.}$$

$$p_g = \frac{2.34}{144} = 0.017$$

or from interaction diagram

$$\frac{N}{f_c A_g} = \frac{55.5}{3 \times 12 \times 12} = 0.96$$

$$\frac{12M_x}{f_c A_g} = \frac{12 \times 19.0}{3 \times 12 \times 12 \times 12} = 0.333$$

$A_{st} = 0.018 \times 144 = 2.59$

Use 4 #9 = 4.0 sq. in.
Problem #2

Data

\( f_y = 40,000 \text{ psi} \)

\( f_s = 16,000 \text{ psi} \)

\( f'_c = 4,000 \text{ psi} \)

\( b = 18'' \)

\( t = 22'' \)

\( g = 0.8 \)

\( M_x = 236 \text{ k.ft.} \)

\( N = 158 \text{ k.} \)

Solution

Compute \( \frac{N}{f'_c A_g} = \frac{158}{4 \times 18 \times 22} = 0.10 \)

From table 26 for \( g = 0.8 \)

\( \frac{p_b}{f'_c A_g} = 0.19 \)

Hence, tension controls. First trial to find area of steel. Assume \( M_{oe} = M \) to get approximate value of area of steel. From Table 26

\( C' = 1.25. \)

\( A_{st} = \frac{C'}{t} \cdot M_{oe} = \frac{1.25}{22} \times 236 \)

\[ = 13.4 \text{ sq. in.} \]
Assume 14 bars of #9 and arranged as shown in sketch.

\[ A_{st} = 14 \text{ sq. in.} \]

\[ p_g = \frac{14}{396} = 0.0354 \]

\[ A_{st1} = 2 \times 5 \times 1.0 = 10 \text{ sq. in.} \]

\[ p_1 = \frac{10}{396} = 0.0253 \]

\[ A_{st2} = 2 \times 2 \times 1.0 = 4 \text{ sq. in.} \]

\[ p_2 = \frac{44}{396} = 0.011 \]

\[ p'y = p_1 + 0.5 p_2 \]

\[ = 0.0253 + 0.5 (0.0101) \]

\[ = 0.0303 \]

\[ K = \frac{A_{s2}}{A_{st1}} = \frac{4}{10} = 0.4 \text{ sq. in.} \]

From Table 34 for \( p_g = 0.035 \): \( g = .8, K = .4: D' = 0.113 \)

Equivalent pure moment

\[ M_{oe} = M_y D' \frac{NT}{12} \]

\[ = 236 - 0.113 \frac{158 \times 22}{12} \]

\[ = 203 \text{ ft. k.} \]

\[ A_s = \frac{C'M_{oe}}{t} = \frac{1.25 \times 203}{22} = 11.53 \text{ sq. in.} \]

\[ p'y = \frac{A_s}{A_g} = \frac{11.53}{396} = 0.0291 \]
Check by interaction diagram.

\[
N' = \frac{N}{f'_{c'g}} = \frac{158}{4 \times 18 \times 22} = 0.10
\]

\[
M'_{y} = \frac{12 M_{y}}{f'_{c'g}} = \frac{12 \times 236}{4 \times 22 \times 396} = 0.081
\]

From diagram 35 b for 4,000/6,000/gy = 0.8

It is noted \( p'y = 0.03 \)

Hence provide \( A_s = 11.53 \text{ sq. in.} \)