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WARPing STRESSES AND DEFLECTIONS
IN CONCRETE PAVEMENTS

By
Manu Gandhi

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March 28, 1973
Date

Arnold Wilson
Chairman, Advisory Committee

Clifford J. Stott
Member, Advisory Committee

James P. Barton
Chairman, Major Department
DEDICATED

To my parents
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Chapter 1

INTRODUCTION

According to Farrell and Patrick(1)* expenditures for all types of surfacing on primary and secondary roads have comprised of about 40 percent of the construction funds as compared with about 25 percent each for grading and structures. In 1970 about 8,000 miles of new concrete pavements were constructed in the United States, [Highway Statics U. S. Bureau of Public Roads, Washington 1970], an increase of approximately 60 percent over that being constructed during the previous five years. Looking into the details, it can be seen that expenditures for concrete pavements represent an important and increasing fraction of the nation's highway investment. The need for improved design criteria is now more pressing than ever.

A deviation or distortion of a slab surface from its original shape usually caused by temperature or moisture differentials, or both for causes shown in Chapter 2 within the slab is called warping.

The occurrence of objectionable roughness of concrete pavements due to uplifted slab-ends was first recognized on the Pacific coast as early as 1922. A similar type of roughness was recognized in the midwestern states in the period 1928 - 1931. It was observed that the severe and frequent distortion of the pavement imparted a

*Numbers in brackets indicate the reference number
rhythmic, rocking, jerking motion to passengers in fast moving traffic which was very objectionable. The distortion ranged in magnitude from those which were negligible to those in which the slab ends were distorted two or more inches from their original position.

Because of the number of projects affected and the severity of the distortions, it became necessary to determine the cause of the distortion (see Chapter 2) and prevention on future methods of maintenance (see Chapter 3). It was also necessary to find out the affects of warping prestressed or reinforced concrete rigid slab, etc.

As early as 1926, Westergaard\(^2\) presented a theoretical analysis of warping stresses due to temperature but their importance had not been generally recognized because of low value of temperature. It remained for Arlington\(^3\) to demonstrate that these warping stresses may be as great as those produced by heavy wheel loads.

A subject of warping stresses and deflections of concrete pavements had been given little attention until now and so there is a corresponding lack of information. This study proves that warping is a relatively simple phenomenon when viewed apart from the complex abstractions that are associated with strength elasticity, external forces, internal forces, etc. Until now questions in many countries (like U.S.A., Germany, Kuwait, etc.), are giving importance to warping stresses and deflections of rigid pavements.

The aim of this report is to determine the causes and prevention of the warping and deflection for rigid concrete pavements. The report also deals about the methods of maintenance, methods of
preventing warping and distortion in new construction. The report is study oriented. Three examples have been solved. The examples deal with the stresses due to warping caused by the effects of temperature, shrinkage and subgrade.
Deformation: upper side warmer

Stress distribution caused by warping

Figure 1. Theoretical Deformation and Stress Diagram of a Pavement Slab Subjected to a Temperature Differential. (10)
Chapter 2

CAUSES OF WARPING STRESSES
AND DISTORTION

Many probable causes for the distortions of rigid pavements are separated into two main groups

(A) Internal forces acting within the slab itself, and
(B) External forces acting on the slab.

(A) Internal Forces Acting Within the Slab

(1) Temperature and Moisture

Warping of the pavement slab is caused by a difference in temperature or moisture of sufficient magnitude between top and bottom of the slab.

Warping due to a difference in moisture content between the top and bottom of the slab results in slab movements so small that they cannot be considered, even if combined with temperature curl as a cause of the objectionable high joints observed in these studies.\(^{(4)}\)

For a given temperature difference between slab surfaces, non-linear temperature or moisture distributions result in larger tensile stresses than linear temperature distribution.\(^{(5)}\)

Diagrammatically indicates the deformations and stress levels caused by a positive temperature gradient. Slabs of three
Figure 2. Supported 17 foot 6 inch slabs (rear) and 6 foot warp slabs at the front (8)
Figure 3. Relation of Plastic Flow Deflection to Ratio of Span Length to Depth (L/D)
different lengths, are shown in Figure 1. The middle picture shows a slab of length equal to a calculable "L critical" which represents the length necessary to have the restrained curling stresses maximized at midlength. Shorter or longer slabs have lesser values at midlength.\(^{(6)}\)

(2) **Shrinkage**

Shrinkage strains due to drying in the concrete member generally are non-uniform and large at the surface and small at the inner portion. Amount and distribution of drying shrinkage in concrete pavements varies with the local atmosphere environment, \(i.e.,\) temperature, humidity, wind\] materials and mix proportions of concrete. Warping stresses in concrete pavement due to shrinkage decreases in the daytime and increases at nighttime. These stresses produce tensile stresses which occur at the surface of the pavements.\(^{(7)}\)

(3) **Plastic Flow of Concrete**

Mr. George W. Washa\(^{(8)}\) found that the plastic flow of concrete affected the concrete slab which depends upon the L/D ratio, slump, etc. Plastic flow is additive to the warp of the slab. The curves of Figure 3 shows that as L/D ratio is increased, plastic flow is increased. See Figure 2 for warp slab at the front is due to the effect of plastic flow of concrete.\(^{(8)}\)

(4) **Volume Changes and Creep**

If concrete were free to deform, any volume changes would be of little consequence, but usually it is restrained by foundations,
Figure 4. Relaxation of Stress Under a Constant Strain of $360 \times 10^{-6}$ (9)
steel reinforcement, or by adjacent concrete. As the potential movement is thus restrained, stresses will be developed which may rupture the concrete. This is particularly true when tension is developed; thus contractions causing tensile stresses are more important than expansion which causes compressive stresses. Differences in moisture contents and thus volume changes of the exposed and unexposed faces of thin concrete slabs, such as highways may cause curling and warping of the slab.

Creep, in general, tends to relieve the stresses in concrete, especially when reinforced. In various structural elements such as slabs or continuous beams, creep relieves some of the stresses in the most highly stressed portions and increases the stress in adjacent portions of the concrete, so that finally stresses are more uniform throughout the member. The relief of high stress as a result of creep is shown (Figure 4) by graph. This relieving of the higher stresses serves to reduce the tendency toward cracking. However, creep may cause objectionable sagging of thin long span slabs under normal drying conditions. The deformation due to creep after loading for 2 years may be 2-4 times the amount of elastic deformation.\(^{(9)}\)

B. External Forces Acting on The Slab

1. Non-uniform Subgrade Soil

The warping is definitely associated with the characteristics of the subgrade soils. Warping is found only with expansive soils e.g. A-4, A-6, A-7 type of soils.\(^{(4)}\)

Warping is caused by swell or shrinkage of the subgrade soils. It is found by observation of the slab and the subgrade soil. Warp-
ing due to shrinkage was considered by Mr. J. R. Shank — 1935 
[ACI Publication - 1935] incidental to his notable treatment of plastic flow. Tests and observation showed high moisture content in the expansive subgrade soil under high joints and showed that the moisture content diminished with increased distance from the joints and cracks or edges of the slab. These moisture gradients resulted from the entrance of water and subsequent absorption by the subgrade at the joints and cracks and at the edges of pavement.

Warping was not caused by placing slabs on subgrade soils of a non-expansive nature. The swelling of the expansive soils produce sufficient force to cause an uplift of the slab ends and edges. (4)

2. **Frost Action.**

Stresses or distortion caused by the expansion was found to be caused by swell of the soil due to absorption of moisture or by frost action. Where frost action played no part, the swell was caused entirely by absorbed moisture. Frost action was found to be the major cause of soil swell in some locations, the slab ends rising during the winter months and receding during the summer months. (4)

3. **Flow or Creep of Subgrade Soil**

Soil flow or creep may or may not be responsible for the warping stress and distortion of the slab. From library research there is no certain indication for creep.

4. **Moving Loads**

Moving loads test on experimental prestressed concrete and
reinforced concrete pavements shows warping stresses at the edges and at corners of the slab. These warping stresses are caused by the loss of subgrade support at the edges of the slab near the corners. Deformation at the corners of prestressed and reinforced concrete slabs were measured to study warping and its effect on the strength of pavement for Kuwait International Airport. \(^{(6)}\)

Warping of the pavement slab due to internal forces within the slab is not enough to cause pavement distortion of the amount found by the experiments, but external forces are responsible for the distortion of the slab, like moving wheel loads.

Evidence of action of external force was found in cracks which developed 10 to 20 feet back of the joint and is caused by flexure of the distorted slabs. These cracks were not always visible on the surface but were at all times definitely wider at the bottom than at the top of the slab. \(^{(4)}\)
Chapter 3

METHODS OF PREVENTING DISTORTION
AND WARPING

Here many probable methods for preventing the distortion or warping of rigid pavements are separated into two main groups: (A) Internal forces acting on the slab, (B) External forces acting on the slab.

A. Internal Forces

(1) Temperature

A temperature difference between the top and bottom of a concrete pavement slab causes a tendency for the slab to curl. This curling is prevented by weight of the slab itself, by using expansion joints or by using insulating materials. The magnitudes of these restrained curling stresses are dependent upon the temperature distribution, concrete properties, underlying support, conditions and edge restraints.

An insulating layer of polystyrene decreases the temperature differentials by about 30 percent and produced non-linear distribution.(10)

(2) Shrinkage

Thermal stresses in concrete pavements due to daily temperature changes should be considered in combination with drying shrinkage strain and thermal strain. According to observed results, warping stresses in concrete pavements due to drying shrinkage causes
tensile stresses [70 to 80 percent of tensile strength of concrete] of the surface of pavements which can be prevented.

Reduction of cement content and use of fly-ash as an additive have been considered as effective in decreasing drying shrinkage of concrete.

The application of expansive concrete on long span concrete pavements such as continuously reinforced concrete pavements and prestressed concrete pavements seems to be effective within the limits of test results to date.\(^2\)

(3) Plastic Flow of Concrete

For slabs of a given w/c ratio, steel ratio, and curing method, the greatest plastic-flow deflections are obtained for the slabs made with the greatest water content and least for those slabs made with the lowest water content. The dry slabs have the greater plastic flow than sealed (covered with paraffin or wax or painting) slabs. The plastic flow deflection also depends upon concrete slump, age, and span. If these factors are increased, then the plastic flow will be increased. The plastic flow deflection can be found out by the following equation.

\[ Y = C_x^{1/a} \]

where

\[ Y = \text{plastic flow deflection in inches} \]

\[ c, a = \text{coefficients for the limiting age (in years)} \]

\[ x = \text{time in days} \]

Plastic flow deflection can be prevented by more effectively
Figure 5. Relation of Plastic flow Deflection to Concrete Slump (8)
sealing the slabs, which are shown by Figure 5. Figure 5 shows that plastic-flow deflections increased as the slump increased, also it shows that the sealed slabs have less deflection than the dry slabs. (6)

(4) **Volume Changes and Creep**

Several factors which may be expected to influence the magnitude of volume changes in concrete are, cement, water contents, composition and fineness of the cement, type and gradation of aggregate admixtures, moisture and temperature conditions, etc. Usually volume changes are restrained by foundations, steel reinforcement or by adjacent concrete subject to different conditions.

The magnitude of the creep depends upon several factors relating to the quality of the concrete such as the aggregate-cement ratio, water cement ratio, kind of aggregates and its grading, admixtures and the age at the time of loading. It also depends upon the intensity and duration of stress. Creep is restrained by controlling the factors which are described above or by steel reinforcement. Steel reinforcement serves to reduce creep-effect by relieving the higher stresses. (9)

B. **External Forces Acting on the Slab**

(1) Subgrade: Detrimental warping of pavement on expansive soil subgrades has been prevented except under unusually severe conditions by construction methods which provided adequate moisture in and proper densification of the subgrade at the time the pavement was placed. This is true regardless of whether the moisture was introduced
artificially or naturally. Moisture was introduced during grading operations or was later introduced by diking, ponding, or sprinkling the subgrade until the water had penetrated to a depth of 18 to 24 inches.

Warping can be prevented by modification of location or grade-line of the road and by grading operations so that non-expansive soils may be used in the subgrade. No general rule has been worked out to determine depth of cover of non-expansive soil over expansive soils necessary to prevent warping.

The use of a bituminous membrane sufficiently protected by a granular base has proved effective. The joints having copper seals of various types or vinyl water stops when properly installed and maintained have been beneficial.

It can be prevented by changing the characteristics of expansive subgrade soils by admixtures of granular materials, powdered materials, such as portland cement, lime and stone dust or by use of bituminous materials.\(^4\)

(2) Frost Action.

The prevalence of high joints due to frost action has been minimized by careful joint maintenance. The results obtained in one location indicate that a special joint sealing material properly applied is effective in preventing leakage of water through joints. The maintenance of contraction joint in a watertight condition has been found to be much easier than that of expansion joints.\(^4\)
Figure 6. Cracking in Concrete Pavement Due to Moving Load. (10)
Figure 7. Plastic Deformation of the Asphalt Concrete Subbase Due to Curling or Warping of the Pavement Slab (10)
(3) **Moving Loads**

From the measurement of the vertical movements of the corners of the three prestressed slabs\(^{(6)}\), it can be said that the warping at the corners of prestressed pavements largely depends on the size of the slab as well as the prestressing forces. The larger the size of the slab and the smaller the prestressing force in both directions, the smaller is the warping at the corner.

The measurement of the vertical movements at the free corner and at the dowelled expansion joint of the reinforced slab is virtually identical. This may well be due to the freedom allowed by the flexible joint filler boards used and the lubricated dowel bars.

Although load tests showed no serious effect of warping, the Michel's opinion is that this warping might cause difficulties during the life of the pavements, especially when fines accumulate under small areas only at the warped corner or edges.\(^{(6)}\)

The heavy wheel-load on the inner surface of the slab may cause stresses that are additive to the restrained stresses in long slabs and produce transverse cracking which are shown in Figure 6.

Deflection measurements were also made under wheel loads placed near the edges. When the slabs were curled downward (upper side warmer), the deflection of the free edges were less than when the slab was flat. Figure 7 illustrates this hypothesis. At the top of the figure is a slab on a rigid base. The middle sketch shows the slab on a yielding base and the bottom sketch shows the inelastic return of the subbase.\(^{(10)}\)

Detrimental warping of pavements on expansive soil subgrades
has been prevented except under unusually severe conditions by construction methods which provided adequate moisture in and proper densification of the subgrade of the time of placing the pavement. This is true regardless of whether the moisture was introduced artificially or naturally.
Chapter 4

"METHODS OF MAINTENANCE" AND "METHODS USED FOR PREVENTING WARPING ON NEW CONSTRUCTION"

Methods of Maintenance

The following is information on the prevention and correction of warping of concrete pavement slabs by maintenance methods.

(1) **Straightening Distorted Pavements by the Addition of Water to the Subgrade**

The states of Kansas and Missouri reported the use of this method as a maintenance operation. It was thought that, light soil swell, due to unequal moisture distribution resulting from water entering the subgrade through joints and along the edges of the slab was the major cause of the distortions. It would be possible to raise the central part of the slab to the same relative elevation as that of the slab ends by artifically introducing water in to the subgrade to equalize the moisture content.

(2) **Maintenance of Joints to Prevent or Minimize Warping**

In Minnesota in the year 1934, 160 miles of distorted pavements were found. They found a decrease of the distortion as a result of special joint sealing during the fall months rather than after freeze-up which had been the general practice in preceding years. Where pavement joints were sealed in the fall months, warping was reduced an appreciable amount, but in all instances where this
maintenance was omitted in the fall, warping was same as during the preceding winter. Spec. 136.03 (soft filler 85-100 pen. at 25°F) with 20 percent celite (i.e. diatomaceous earth) was used on projects constructed in 1933 and for special maintenance on older projects. It was a most satisfactory filler.

(3) Correcting Warped Slabs by Mudjacking

For the correction of distorted pavement slabs the only effective method employed in Missouri has been mudjacking. Accurate measurements of profile elevations on slabs straightened by this method have shown a satisfactory retention of vertical alignment for periods greater than one year. Considerable skill is required in administering the treatment in order that excessive breaking of the slabs may not occur. The cost of mudjacking depends somewhat upon the length of section requiring straightening, but for conditions in Missouri, it is estimated at 12 to 15 cents per square yard.

METHODS USED FOR PREVENTING WARping ON NEW CONSTRUCTION

After it became evident that warping was caused by differential swelling of the subgrade soil due to absorption of water, efforts were made to prevent its occurrence on new construction. Inasmuch as the design and construction of the pavement slab itself had little effect on the occurrence of warping, attention was given to the design and construction of joint drains and watertight joints, and to control of the swell of the subgrade soil by controlling soil moisture and density at the time of concreting or by blanking the subgrade with a protective material. The results of effects to
prevent warping on new construction by these methods are described here.

(1) Joint Fillers and Seals

The many different materials and mixtures of joint fillers and seal materials used on new construction, and their effectiveness has been described previously in this project.

(2) Joint Drains

Between leaky expansion joints and distortion of slab ends there can be provided drains to collect water entering leaky joints. We can use ordinary blind under-drain (which are deep back-filled with crushed rock) tile drains laid in a concrete trough or porous concrete drains.

(3) Subgrade Paper

Subgrade paper was found to adhere to the pavements and to break at joints and did not prevent infiltration of surface water through joints and crack to the subgrade.

(4) Bituminous Membrane Under Granular Bases

In California it has been found that an asphalactic membrane covered with a blanket of 9 to 18 inches of non-swelling imported borrow has given satisfactory results. An analysis of 25 projects constructed between 1933 to 1937, using a bituminous membrane of 0.5 to 0.7 gallons per square yard of 'E' grade asphalt below a blanket of granular soil showed no distortion of the pavement. The use of asphalactic membrane has not been generally continued because of construction difficulties.

(5) Moisture and Density Control of Subgrade Soil
In this method water will be used before the construction of pavement. In Texas on projects in Patter and Carson counties, there exists a uniform type clay soil. The subgrade was wetted with 6,000 gallon of water per 100 feet directly. It was found that in approximately three-years, the subgrade soil throughout the projects arrived at approximately the same moisture content. This method can be used when the subgrade is uniform, and highly expansive clay soil. A survey was made in 1939 on thirty projects in eastern and central Kansas, constructed during 1937-1938. Among the above thirty projects, twenty-three projects were entirely free of warping. This method is not effective for highly expansive soil because the volume change cannot be controlled within safe limits.

(6) **Selected Low-value Change Base Course**

The type of base course and the function which it was expected to perform in preventing warping differed from project to project. In this case, materials used were of a granular and non-expansive soils type. Nevertheless, some base courses were ineffective in preventing warping while others were used successfully. The thickness of the base courses and type of subgrade were ineffective in preventing warping. The thin base courses are effective for the expansive soil subgrade. Thick base courses are effective for non-expansive soils. There is no indication what thickness is needed for different types of soil, but 9 to 18 inches thick base courses of non-expansive soil are necessary.
(7) **By Changing the Characteristics of Soil**

In this method, warping can be prevented by changing the characteristics of expansive soils to those of non-expansive soils. We can change the characteristic of expansive soils by oil treated or tar treated subgrade.
Chapter 5

EXAMPLES

The effects of temperature and shrinkage in warping of slab can be illustrated by the following numerical examples.

NOTATION

(A)  $E$  Modulus of elasticity, kg/cm$^2$ (PSI)
$\alpha$  Coefficient of temperature expansion l/c 1/F
$tm$  temperature in the center of cross-section c
to  temperature on the upper side of the slab c
tu  temperature on the lower side of the slab c

$\Delta t = \frac{to-tu}{d}$  temperature gradient. C/cm

d  thickness of slab, cm.

$L$  slab length (distance between two joints) cm.
$l$  theoretical slab length cm.

$l_{\text{crit.}}$  theoretical slab length to maximize the stress at mid-length, cm.

$\delta \omega$  undisturbed warping stress Kg/cm$^2$
$\delta' \omega$  disturbed warping stress Kg/cm$^2$
$\delta'' \omega$  decreased warping stress ($l < 0.9 l_{\text{crit}}$

$\gamma$  poisson's ratio

(B)

$\Delta$  displacement
\( \Delta \) displacement

\( L \) Length

\( R \) Radius of curvature

\( K \) Curvature in millionths of radians per inch

\( S_c \) Concrete strain after banding, in millionths

\( S_s \) Steel strain in millionths

\( d \) Effective depth -- inches

\( t \) Total depth -- inches

\( e \) Eccentricity -- inches

\( (c) \)

\( K \) Modulus of subgrade reaction PCI (pounds cubic inch)

\( E \) Modulus of Elasticity psi

\( \Delta t \) Temp change F/inches

\( C_1 \) Coefficients in any dissection

\( C_2 \) Coefficients in perpendicular or to \( C_1 \) direction

\( \varepsilon_t \) Elastic strength in/in

\( \Delta t \) Temp. change F/inches

\( \mu \) Poisson ratio

\( +V_e \) Indicates tensile stresses psi

\( -V_e \) Indicates compressive stresses psi
(a) Effect of Temperature on Concrete

In these examples theoretical calculations of the slab was analyzed as both a beam and a plate.

(1) Beam (Narrow slab) Analysis

Critical slab length regarding poisson's ratio (r = 1/6)

\[ \lambda_{crit} = 20 \times d \times \sqrt{\alpha \times \Delta \times E} \]

For \( E = 3000,000 \text{ Kg/cm}^2 \)
\( \Delta t = 0.9 \text{ c/cm} \)
\( \alpha = 10^{-5} \text{ I/C} \)
\( d = 22 \text{ cm} \)
\( L = 7 \text{ meters} \)

becomes \( \lambda_{crit} = 33 \times d \)

warping stress in \( \text{Kg/cm}^2 \)

\[ \delta_\omega = \frac{1}{1-\nu} \times \frac{d}{2} \times \Delta t \times \alpha \times E \quad \text{for } \lambda > 1.1 \lambda_{crit} \]

\[ \delta'_\omega = 1.2 \times 8\delta_\omega \quad \lambda = \lambda_{crit} \]

\[ \delta''_\omega = \left( \frac{1-40}{0.9 \lambda_{crit}} \right)^2 \delta_\omega \quad \lambda < 0.9\lambda_{crit} \]

The values above \( E, \Delta t, \alpha \) given

Substitute these values

\[ \delta_\omega = 1.63 \times d = 36 \text{ Kg/cm}^2 \]
\[ \delta'_\omega = 1.95 \times d = 43 \text{ Kg/cm}^2 \]
\[ \delta''_\omega = \frac{1}{545 \times d} (L - 40)^2 \text{ Kg/cm}^2 \]

(b) Quadratic Slab

Critical slab length = \( \lambda_{crit} \times 22.8 \times d \times \alpha \times \Delta t \times E \)

For the mentioned values above becomes:
\( \ell \text{ crit.} = 37 \times d = 814 \text{ cms.} \)

Warping stress in Kg/cm\(^2\)

\[ \delta_\omega = 1.63 \times d = 36 \text{ Kg/cm}^2 \]

\[ \delta'_\omega = 1.95 \times d = 43 \text{ Kg/cm}^2 \]

\[ \delta''_\omega = \frac{1}{680 \times d} (L - 40)^2 \text{ Kg/cm}^2 \]

\[ L = \ell + 40 \text{ in joint distance in cms} \]

\[ d = \text{ is thickness of the slab in cms.} \]

(B) Warping as a function of shrinkage\(^{14}\)

(1) Calculate the probable midspan warping deflection due to shrinkage of a 6 inch slab width \( d = 5 \text{ feet} \) \( L = 10 \text{ feet} \) with medium shrinkage and \#4 bars at 6 inches.

Equation:

\[ K = \frac{S_c}{d} \left( 1 - \frac{S_s}{S_c} \right) \]

\[ S_c = 75 \text{ percent of total shrinkage} = 0.75 \times 600 \]

\[ = 450 \text{ millionths.} \]

\[ \frac{S_s}{S_c} = 0.30 \text{ (from graph)} \]

\[ K = \frac{450}{5} (1 - 0.30) \text{ radians per inch} = 63 \text{ millionths} \]

\[ \Delta = \frac{KL^2}{8} \times \frac{63(14,400)}{8(1,000,000)} = 0.1134 \text{ inches} \]

If steel is exposed on the soffit face,

\[ d = 5.75 \text{ inches} \]

\[ \frac{S_s}{S_c} = 0 \]

\[ K = \frac{450}{5.75} (1 - 0) \text{ radians per inch} = 78.2 \text{ millionths} \]


\[ \Delta = \frac{78.2(14,400)}{8(1,000,000)} = 0.141 \text{ inches} \]

\[ \Delta = 0.141 \text{ inches} \]

Observations indicate that an exposed bar even when bonded to adjacent concrete is negligibly affected, therefore \( \frac{S_s}{S_c} = 0 \)

(C) A Rigid Pavement Has the Following Design

a. Slab thickness, \( h = 9'' \), slab dimensions 40 feet (contraction joints) by 24 feet (12 foot lanes, expansion joints)

b. Tie bars ar longitudinal joint 1/2 inch round by 36 inches long, spaced 30 inches c-c.

c. Longitudinal distributed steel. No. 3 wires spaced at 6 inches, diameter of wire 0.2437 inches.

d. Traverse distributed steel No. 4 wires spaced at 12 inches and diameter of wire 0.2253 inches.

e. Dowel bars, 3/4 inch round by 18 inches long, spaced 12 inches on centers, first dowel 6 inches from edge.

f. Modulus of subgrade reaction, 100 psi

Determine the following stresses in the pavement for a 12,000 pound dual wheel, the pressure 75 psi dual spacing 13.5 inches.

a. Calculate the warping stress for a 12 foot wide slab with various lengths assuming temperature differential to be day +3.0°F per inch of slab thickness, and night -1.0°F per inch of slab thickness. Plot a curve of stress vs distance.
Solution:

(a) All formula. Coefficients used from "principles of pavement design by yoder (17) from the topic stresses in rigid pavements.

From table 3.1, page 55

\[ h = 9 \quad K^* = 100 \text{ psi} \]

\[ \ell = 39.71 \text{ inches} \]

\[ L_x = 40 \times 12 = 480 \text{ inches} \]

\[ L_y = 12 \times 12 = 144 \text{ inches} \]

\[ \frac{L_x}{\ell} = \frac{480}{39.71} = 12.08; \quad \frac{L_y}{\ell} = \frac{144}{39.71} = 3.625 \]

The warping stresses coefficients from Figure 3.3, page 58

\[ c_x = 1.02 \]

\[ c_y = 0.40 \]

Take, \( \mu = 0.15 \quad E = 4 \times 10^6 \text{ lb/in}^2 \quad \epsilon_r = 5 \times 10^{-6} \)

Sample calculations

**Interior Stresses (6 \_1)**

\[ b = \frac{E \epsilon_r \times \Delta t}{2} \left( \frac{C_1 + C_2}{1 - \mu^2} \right) \]

\[ \Delta t = + 3.0^\circ \text{F/inches for day time} \]

\[ \Delta t \_\text{day} = \frac{4 \times 10^6 \times 5 \times 10^6 \times 3 \times 9}{2} \left( \frac{1.02 \times 0.15 \times 0.4}{1 - 0.0225} \right) \]

\[ = 270 \times \frac{1.08}{0.9775} = 298 \text{ psi} \]

\[ b \_\text{day} = 298 \text{ psi} \]

\[ \Delta t \_\text{night} = \frac{4 \times 10^6 \times 5 \times 10^6 \times (-1) \times 9}{2} \times \frac{1.08}{0.9775} \]

\[ = -90 \times \frac{1.08}{0.9775} = -99.5 \text{ psi} \]

\[ b \_\text{night} = -99.5 \text{ psi} \]
Edge Stresses ($\sigma_e$)

$$
\sigma = \frac{C \cdot E \cdot e \cdot \Delta t}{2}
$$

$$
\sigma_{\text{day}} = \frac{1.02 \times 4 \times 10^6 \times 5 \times 10^6 \times 3 \times 9}{2} = 275.4 \text{ psi}
$$

$$
\sigma_{\text{day}} = 275.4 \text{ psi}
$$

$$
\sigma_{\text{night}} = \frac{1.02 \times 4 \times 10^6 \times 5 \times 10^6 \times (-1) \times 9}{2} = -91.8 \text{ psi}
$$

$$
\sigma_{\text{night}} = -91.8 \text{ psi}
$$
Table 1.
Variation of the Warping Stress for a 12-Foot Slab Width Along The Length of the Slab

<table>
<thead>
<tr>
<th>X (psi)</th>
<th>L_x (feet)</th>
<th>( \frac{L_y}{x} ) (inches)</th>
<th>( \frac{L_y}{x} )</th>
<th>C_1</th>
<th>C_2</th>
<th>( b_1 ) (day psi)</th>
<th>( b_1 ) (night psi)</th>
<th>( b_e ) (day psi)</th>
<th>( b_e ) (night psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>10</td>
<td>39.71</td>
<td>3.02</td>
<td>3.63</td>
<td>0.16</td>
<td>0.40</td>
<td>58.2</td>
<td>19.4</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>20</td>
<td>39.71</td>
<td>6.04</td>
<td>3.63</td>
<td>0.90</td>
<td>0.40</td>
<td>262</td>
<td>87.4</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>30</td>
<td>39.71</td>
<td>9.06</td>
<td>3.63</td>
<td>1.05</td>
<td>0.40</td>
<td>306.5</td>
<td>102.0</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>40</td>
<td>39.71</td>
<td>12.08</td>
<td>3.63</td>
<td>1.02</td>
<td>0.40</td>
<td>298</td>
<td>99.5</td>
</tr>
</tbody>
</table>

*Here Modulus Subgrade Reaction 100 lb/cubic in. which is fair value (fair condition for pavements)
"Variation of the warping stress for a 12-foot slab width along the length of the slab."

Stresses vs Distances

Scale
X axis 1" = 50
Y axis 1" = 10 feet
(b) Calculate warping stresses for various slab widths, plot a curve of stress vs. distance \( L_y \) varies from 0' to 12', \( L_x = 40' \) (constant)

**Solution** (Sample Calculations)

\[
\frac{L_y}{\xi} = \frac{144}{39.71} = 2.625 \quad \text{coefficients from graph p. 58}\]

\[
\frac{\xi}{\xi} = \frac{480}{39.71} = 12.08 \quad c_1 = 0.40 \]

\[
c_2 = 1.02
\]

\[
6_{\text{day}} = \frac{4 \times 10^6 \times 5 \times 10^{-6} \times 3 \times 9}{2} \frac{(1.02 + 0.15 \times 0.40)}{(1-0.0225)} = 153 \text{ psi}
\]

\[
6_{\text{day}} = 153 \text{ psi}
\]

\[
6_{\text{night}} = \frac{4 \times 10^6 \times 5 \times 10^{-6} \times (-1) \times 9}{2} \frac{(0.40 + 1.02 \times 0.15)}{1-0.0225} = 51 \text{ psi}
\]

\[
6_{\text{day}} = \frac{0.40 \times 4 \times 10^6 \times 5 \times 10^{-6} \times 3 \times 3}{2} = 108 \text{ psi}
\]

\[
6_{\text{night}} = \frac{0.4 \times 4 \times 10^6 \times 5 \times 10^{-6} \times (-1) \times 9}{2} = 36 \text{ psi}
\]
<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>K (psi)</th>
<th>Ly (feet)</th>
<th>L_y/\lambda</th>
<th>L_x/\lambda</th>
<th>C_1</th>
<th>C_2</th>
<th>\sigma_1 (psi)</th>
<th>\sigma_1 (psi)</th>
<th>\sigma_6 (psi)</th>
<th>\sigma_6 (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>3</td>
<td>0.906</td>
<td>12.1</td>
<td>0.0</td>
<td>1.02</td>
<td>42.3</td>
<td>14.10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>6</td>
<td>1.812</td>
<td>12.1</td>
<td>0.04</td>
<td>1.02</td>
<td>50.6</td>
<td>17.8</td>
<td>9.4</td>
<td>3.1</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>9</td>
<td>2.718</td>
<td>12.1</td>
<td>0.10</td>
<td>1.02</td>
<td>70.0</td>
<td>27.0</td>
<td>23.3</td>
<td>9.0</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>12</td>
<td>3.624</td>
<td>12.1</td>
<td>0.40</td>
<td>1.02</td>
<td>153.0</td>
<td>51.0</td>
<td>108</td>
<td>36</td>
</tr>
</tbody>
</table>
Variation of the warping stresses for a 12 foot slab width along the width of the slab

Distance vs Stresses

Scale
X axis
24 unit = 3 feet
Y axis
20 unit = 50 psi

Distance (ft) →

Stress (psi) ↑

σ₁ day
σ₂ day
σ₁ night
σ₂ night
Chapter 6

CONCLUSIONS

The many probable causes for the distortion of concrete pavements which are advances, are separated into two main groups.

A. Internal Forces Acting Within the Slab Itself Due To
   (1) Vertical temperature or moisture differential in the slab
   (2) Shrinkage strains due to drying of the concrete
   (3) Plastic flow of concrete
   (4) Volume changes and creep in the concrete slab

B. External Forces Acting on the Slab Due To
   (1) Non-uniform soil-shrinkage caused by loss of moisture from the subgrade soil.
   (2) Non-uniform soil swell caused by frost action
   (3) Moving cyclic loads on the concrete pavements.

C. The combination of (1) Internal forces acting within the slab itself and (2) External forces acting on the slab are the causes for warping and distortion.

   The warping stresses and deflection can be prevented in the following ways.

A. For Internal Forces
   (1) Insulating layer of polystyrene decrease temperature differentials.
(2) Shrinkage strains can be prevented by cement content, use of fly ash, and use of reinforcement bar.

(3) Warping due to plastic flow can be prevented by sealed slab, decreasing by $l/\delta$ ratio.

(4) Control of the volume changes and creep by attending the factors like W/C ratio, admixtures, steel reinforcement, etc.

B. External Forces

(1) Soil shrinkage can be prevented by changing the modification of location or grade line of the road, and by use of various joints, or by a use of bituminous membrane.

(2) Distortion of the slab at the edges or corner due to frost action can be prevented by using careful joint maintenance.

(3) Moving loads. There is no other particular indication to prevent warping stresses or deflection for moving loads.

A longer crack free slab may be obtained when internal forces acting within the slab itself due to temperature are reduced 30 percent by using polystyrene.

Methods for correcting of warping of concrete pavements (1) mudjacking, carried on with proper precautions, is an effective method of straightening warped slabs. It is most economical method and (2) the introduction of water into the soil by artificial means has been partly successful in the correction of high joints.

Warping stresses and deflection are caused by the external forces like subgrade conditions, frost action, moving load, etc. We can present the internal forces by calculations and experiments
at the time of construction and also by satisfactory designing the concrete mixture. Internal forces are only responsible for the warping stresses and distortion of the slab.
REFERENCES

1. Farrel and Patrick, Highway Research Board Proceedings, 32:1, 1953


17. Yoder *Principles of Pavement Design* by Yoder.

18. *ACI Publication Sp-20*, "Causes, Mechanism, and Control of Cracking in Concrete."